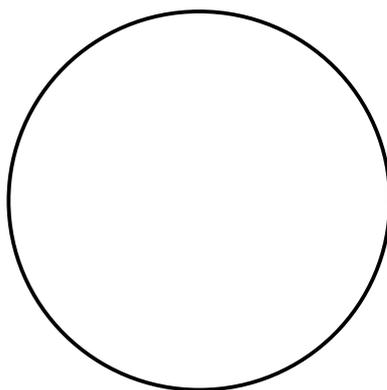


Easy as π

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NAME: _____

1. Suppose we draw a circle of radius 1 inch (it doesn't matter what units we use—inches, centimeters, meters, etc., but I'll stick to inches). In fact, I've drawn such a circle below with my compass:



- (a) What is the diameter of the circle? (Hint: no need to measure!) _____
 - (b) Estimate the circumference of the circle: _____
This question isn't so easy, is it? One way to estimate the circumference is to wrap a string around the circle, then measure the string. Try that approach (I'll supply the string).
 - (c) Divide the circumference of the circle by the diameter. Write the answer you get here: _____
2. It turns out that if you measure the circumference of a circle and divide by the diameter of the circle, you'll always get the same number. Take the "circle" I give you (a can) and measure its diameter. Then wrap a string around the can and measure the length of the string (this will be the circumference of the circle). Divide the circumference by the diameter and write your answer here: _____
 3. The answer to the last question (and question 1(c)) should be a number that's a bit bigger than 3, depending on how carefully you measure. If you make the measurements **VERY CAREFULLY** (to within a hundredth of inch), your answer would be very close to 3.14 (but not exactly equal to 3.14). This number—the circumference of a circle divided by its diameter—is called "pi" (pronounced "pie"). Mathematicians use the Greek letter π for this number.

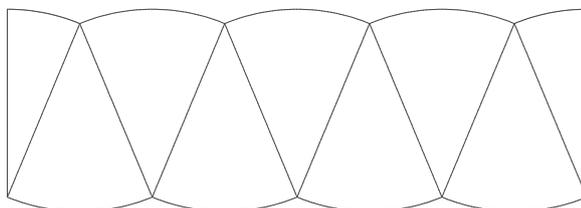
If you could carry out measurements **EXACTLY**, you'd find that π is about equal to

$$\pi = 3.1415926535897932384626433832795028841971693993751058209749445923078 \dots$$

I use dots because the decimal expansion of π NEVER ends, and in fact, has no known pattern (kind of like the prime numbers!) Of course, mathematicians don't compute the decimal expansion of π by wrapping strings around cans; there are purely mathematical ways for computing π , but we won't go into them right now.

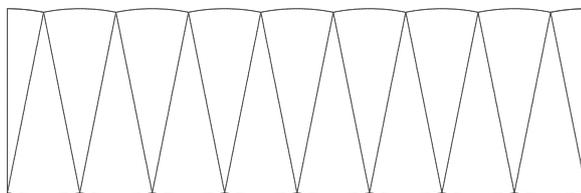
Here's some interesting facts about π :

- The decimal expansion for π has been computed to 1,241,100,000,000 digits, in December 2002. If you wrote this decimal expansion out (like I did above, though I only gave 68 digits) it would be about one and half million miles long!
 - On October 3, 2006, Akira Haraguchi of Japan recited the first 100,000 digits of π from memory! It took him 16 hours (with five minute breaks every two hours).
 - π CANNOT be written as a fraction with whole numbers for the numerator and denominator. The fraction $\frac{22}{7}$ (or $3\frac{1}{7}$) is pretty close to π , but not exact. The fraction $\frac{355}{113}$ (or $3\frac{16}{113}$) is even closer. These two fractions were found by a Chinese mathematician, Zu Chongzhi, in the 5th century.
 - In the Star Trek episode "Wolf in the Fold", Mr. Spock drives an evil intelligence out of the Enterprise's main computer by asking the computer to determine, to the last digit, the decimal expansion of π .
4. The number π also pops up when you compute the area of a circle. Look at the circle of radius 1 inch at the start of the handout. I claim that the area of this circle is exactly π square inches! To see why, take the paper circle I give you (it has radius 1 inch) and cut it up into "pizza slices". I've marked the circles so they're easy to cut into eight pieces; cut one of the 8 pieces in half again, and arrange the pieces like below:



The slices form an approximate rectangle whose area equals the area of the circle. Find the area of the approximate rectangle, by measuring its length and height.

If we did this with more slices, say 16, the picture would look like



Do you see why the height of the approximate rectangle is very nearly equal to 1 and the length is very nearly equal to π ? Thus the area is $1 \times \pi$, which is π !