Lecture 3-1

Mobile Robot Kinematics

Course Announcements

- Tuesday's Quiz on **Kinematics** will be open book, open notes, open neighbor (highly theoretical)
- Bring a calculator for Tuesday's Quiz
- Bring your laptop and robot everyday
- Lab 2 demo due Thursday, 3/26/09
- Lab 2 memo and code due by midnight on Friday, 3/27/09
- Upload memo and code to Angel
- Memo and Code grades on Angel by Thursday

Quote of the Week

"Just as some newborn race of superintelligent robots are about to consume all humanity, our dear old species will likely be saved by a Windows crash. The poor robots will linger pathetically, begging us to reboot them, even though they'll know it would do no good."

Anonymous

Mobile Robot Kinematics

- ***Mobile Robot Kinematics*** is the dynamic model of how a mobile robot behaves
- Kinematics is a description of mechanical behavior of the robot for design and control
- Mobile Robot Kinematics is used for:
  - Position estimation
  - Motion estimation

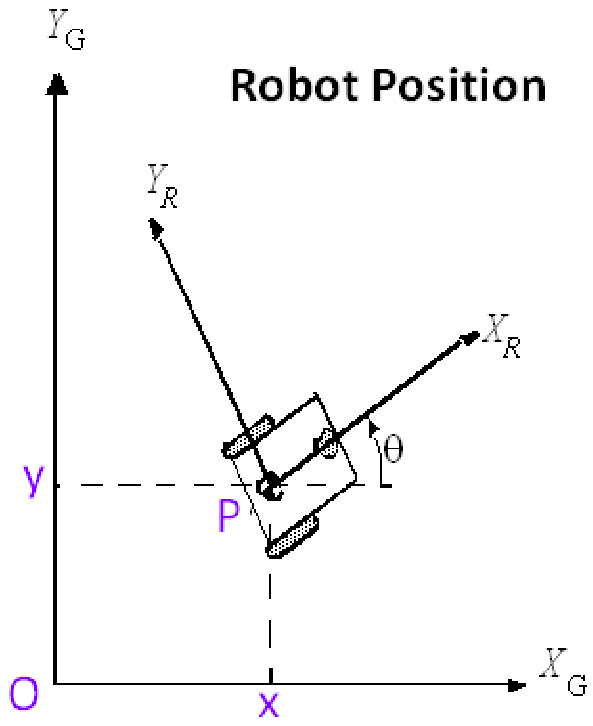
Mobile Robot Kinematics cont.

- Mobile robots move unbounded with respect to their environment
  - There is no direct way to measure robot position
  - Position must be integrated over time from velocity ($v = dp/dt$)
  - The integration leads to inaccuracies in position and motion estimation
- Each wheel contributes to the robot's motion and imposes constraints on the robot's motion
- All of the constraints must be expressed with respect to the reference frame (global inertial frame)

Forward versus Inverse Kinematics

- **Forward Kinematics** involves estimating a mobile robot’s motion and /or pose given the angular and linear velocity
- **Inverse Kinematics** involves determining the robot’s angular and linear velocity to achieve a given robot motion and/or pose

Robot Reference Frame



$$^{G}P_{R} = \begin{bmatrix} x & y & \theta \end{bmatrix}^{T}$$

- The robot's reference frame is three dimensional including position on the plane and the orientation, **$\{X_R, Y_R, \theta\}$**
- The axes **$\{X_G, Y_G\}$** define the inertial global reference frame with origin, **O**
- The angular difference between the global and reference frames is **θ**
- Point **P** on the robot chassis in the global reference frame is specified by coordinates **(x, y)**
- The vector **$^{G}P_{R}$** describes the location of the robot with respect to the inertial global reference frame.

Orthogonal Rotation Matrix

- The **orthogonal rotation matrix** is used to map motion in the global reference frame $\{X_G, Y_G\}$ to motion in the robot's local reference frame $\{X_R, Y_R\}$
- The **orthogonal rotation matrix** is used to convert robot velocity in the global reference frame $\{X_G, Y_G\}$ to components of motion along the robot's local axes $\{X_R, Y_R\}$
- The vector P_R describes the location of the robot with respect to the local reference reference frame.

$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

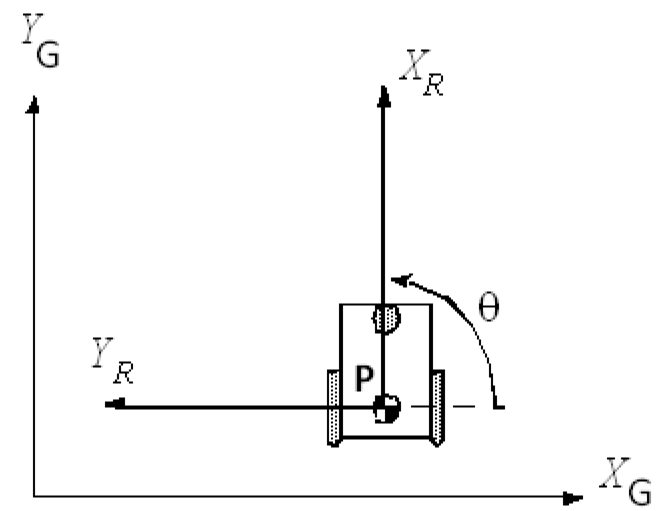
$$\dot{P}_R = R(\theta)\,{}^G\dot{P}_R = R(\theta)\begin{bmatrix} \dot{x} & \dot{y} & \dot{\theta} \end{bmatrix}^T$$

Rotation Example:
(global to local reference frame)

- Suppose that a robot is at point **P** and $\theta = \pi / 2$ and the robot's velocity with respect to the global reference frame is $(\dot{x}, \dot{y}, \dot{\theta})$
- Find the robot's motion with respect to the local reference frame

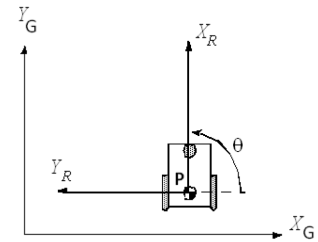
  $\{X_R, Y_R\}$

- The motion along **X_R** and **Y_R** due to θ is



$$\dot{P}_R = R\left(\frac{\pi}{2}\right) {}^G\dot{P}_R = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \dot{y} \\ -\dot{x} \\ \dot{\theta} \end{bmatrix}$$

Rotation Example:
(local to global reference frame)



- Now suppose that a robot is at point **P** and $\theta = \pi/2$ and the robot's velocity with respect to its local frame is $(\dot{x}, \dot{y}, \dot{\theta})$
- Find the robot's motion in the global reference frame $\{X_G, Y_G\}$
- The motion along X_G and Y_G due to θ is

$$ {}^{G}\dot{P}_R = R(\theta)^{-1}\,\dot{P}_R $$

$$ {}^{G}\dot{P}_R = R\left(\frac{\pi}{2}\right)^{-1} V_R = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} -\dot{y} \\ \dot{x} \\ \dot{\theta} \end{bmatrix} $$

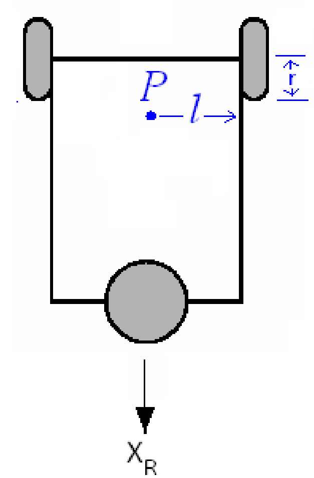
Forward Kinematics Model

- ***Forward Kinematics*** provides an estimate of the robot's position given its geometry and speed of its wheels
- It requires accurate measurement of the wheel velocities over time
- However, position error (accumulation error) grows with time

Forward Kinematics Model: Differential Drive Robot

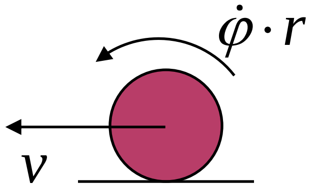
- Consider a differential drive robot which has 2 wheels with radius **r**, a point **P** centered between the 2 drive wheels and each wheel is a distance ***ℓ*** from **P**
- If the rotational speed of the 2 wheels is $\dot{\varphi}_1$ and $\dot{\varphi}_2$ then the forward kinematic model is

$$^{G}\dot{P}_R = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = f(l, r, \theta, \dot{\varphi}_1, \dot{\varphi}_2)$$



Forward Kinematics Model: Differential Drive Robot Linear Velocity

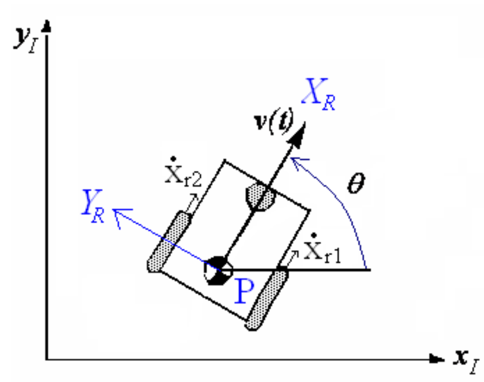
- To find the **linear velocity** in the direction of **$+X_R$** each wheel contributes one half of the total speed.



$$\dot{x}_{r1} = (1/2) r \dot{\varphi}_1 \qquad \dot{x}_{r2} = (1/2) r \dot{\varphi}_2$$

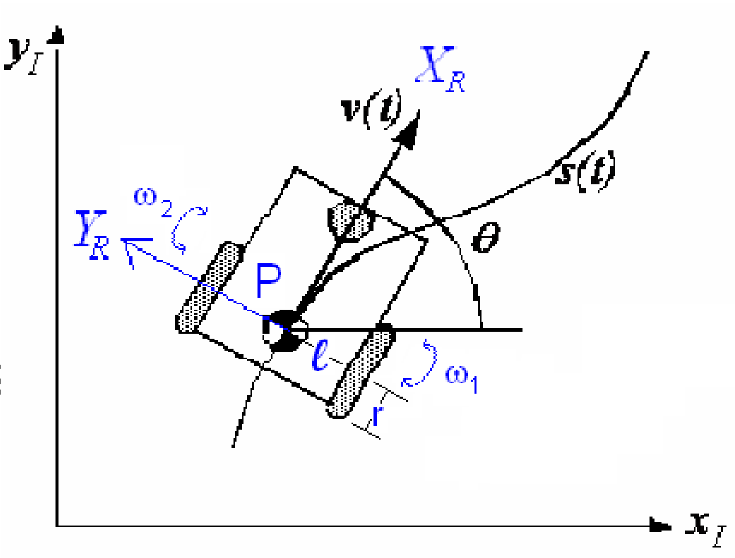
$$v(t) = \dot{x}_{r1} + \dot{x}_{r2}$$

- Since the wheels cannot move sideways, the velocity in the direction of **Y_R** is zero.



Forward Kinematics Model: Differential Drive Robot Angular Velocity

- The **angular velocity** about θ is calculated from the contribution from each of the two wheels working alone.
- The right wheel contributes counterclockwise rotation ω_1 around the left wheel.
- The left wheel contributes clockwise rotation ω_2 about the right wheel.
- Each rotation has a radius of 2ℓ.



$$\omega_1 = \frac{r\dot{\varphi}_1}{2l}$$

$$\omega_2 = -\frac{r\dot{\varphi}_2}{2l}$$

Complete Forward Kinematics Model: Differential Drive Robot

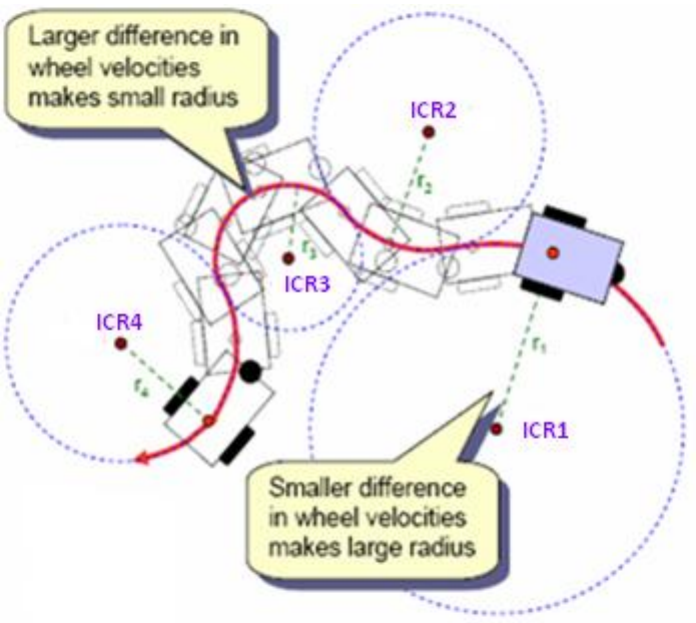
Given the robot's rotation with respect to the global reference frame, wheel velocities, radius of the wheels and distance between the wheels it is possible to find the robot's velocity with respect to the global reference frame. The **complete forward kinematic model** is

$$^{G}\dot{P}_{R} = R(\theta)^{-1}\dot{P}_{R} = R(\theta)^{-1}\begin{bmatrix} \dot{x}_{r} \\ \dot{y}_{r} \\ \dot{\theta}_{r} \end{bmatrix}$$

$$^{G}\dot{P}_{R} = R(\theta)^{-1}\begin{bmatrix} \dot{x}_{r1} + \dot{x}_{r2} \\ 0 \\ \omega_{1} + \omega_{2} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \frac{r\dot{\varphi}_{1}}{2} + \frac{r\dot{\varphi}_{2}}{2} \\ 0 \\ \frac{r\dot{\varphi}_{1}}{2l} - \frac{r\dot{\varphi}_{2}}{2l} \end{bmatrix}$$

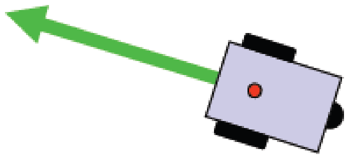
Instantaneous Center of Rotation (ICR)

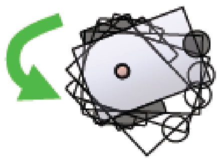
- The ICR has a *zero motion line* drawn through the horizontal axis perpendicular to the wheel plane
- The wheel moves along a radius R with center on the zero motion line, the center of the circle is the *ICR*
- *ICR* is the point around which each wheel of the robot makes a circular course
- The *ICR* changes over time as a function of the individual wheel velocities



Instantaneous Center of Rotation (ICR) cont.

- When R is infinity, wheel velocities are equivalent and the robot moves in a straight line
- When R is zero, wheel velocities are the negatives of each other and the robot spins in place
- All other cases, R is finite and non-zero and the robot follows a curved trajectory about a point which is a distance R from the robot's center
- Note that differential drive robot's are very sensitive to the velocity differences between the two wheels…making it hard to move in a perfectly straight line

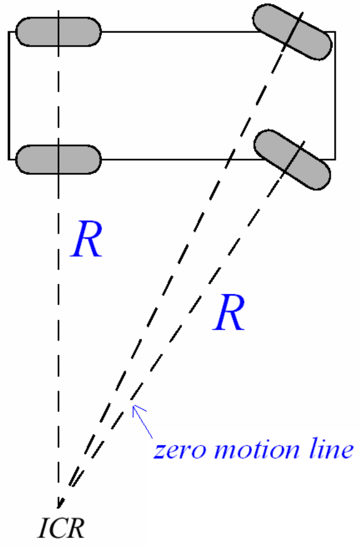




Degree of Mobility

- The ***degree of mobility*** quantifies the degrees of controllable freedom of a mobile robot based on changes to wheel velocity

- The kinematic constraints of a robot with respect to the degree of mobility can be demonstrated geometrically by using the ***ICR***



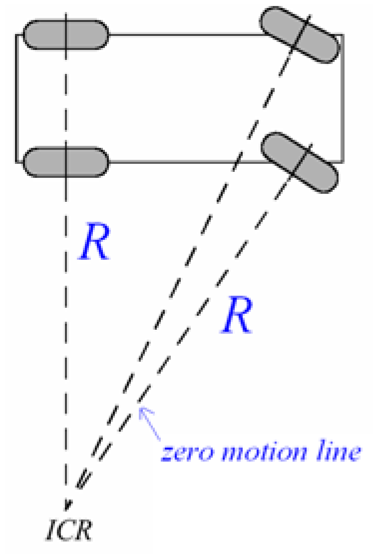
Degree of Mobility

- *Robot mobility* is the ability of a robot chassis to directly move in the environment
- The *degree of mobility* quantifies the degrees of controllable freedom based on changes to wheel velocity
- *Robot mobility* is a function of the number of constraints on the robot's motion, not the number of wheels

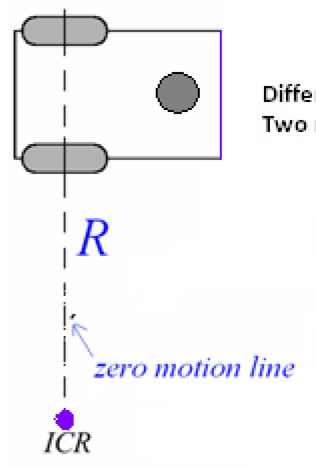
Differential Drive Robot Degree of Mobility

- The Ackerman vehicle has two independent kinematic constraints because all of the zero motion lines meet at a single point. There is one single solution for robot motion.

- A differential drive robot has one independent kinematic constraint because both of the zero motion lines are aligned along the same horizontal line . There are infinite solutions for robot motion. The castor wheel imposes no additional kinematic constraint



Four wheel robot with Ackerman steering

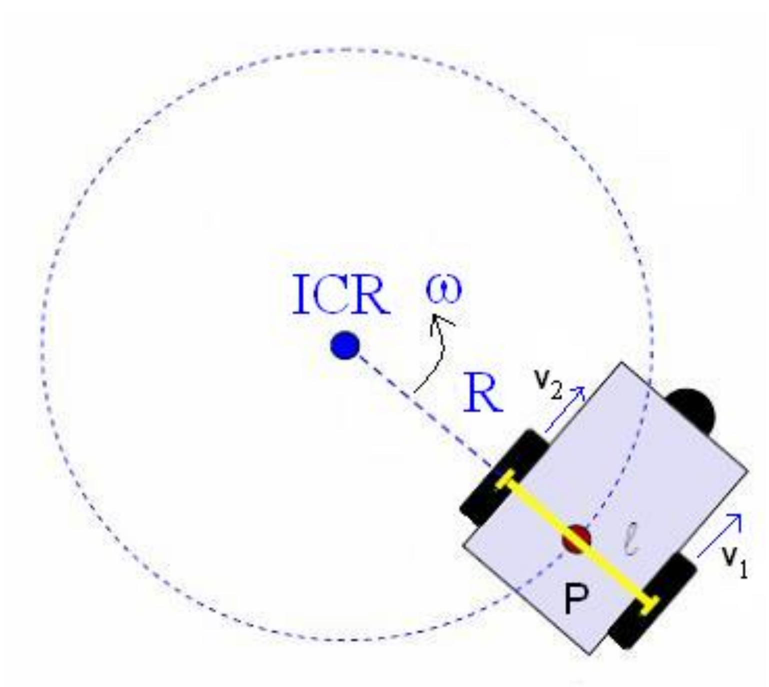


Differential Drive Robot
Two motorized wheels and castor

Forward Kinematics

Assume that at each instance of time, the robot is following the **ICR** with radius **R** at angular rate **ω**

$$\omega = \frac{(v_1 - v_2)}{2l} \qquad R = \frac{V}{\omega} = \frac{l(v_1 + v_2)}{(v_1 - v_2)}$$



V = robot forward velocity

v_1 – right wheel velocity

v_2 – left wheel velocity

ω - robot angular velocity

ℓ – distance from robot center to wheel

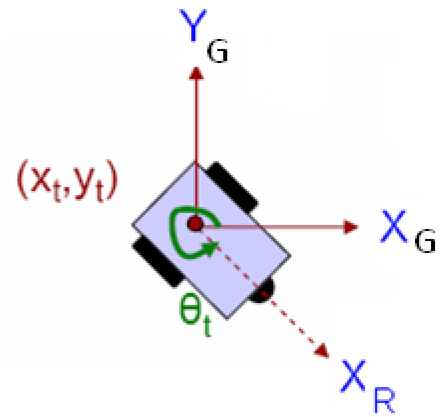
Forward Kinematics cont.

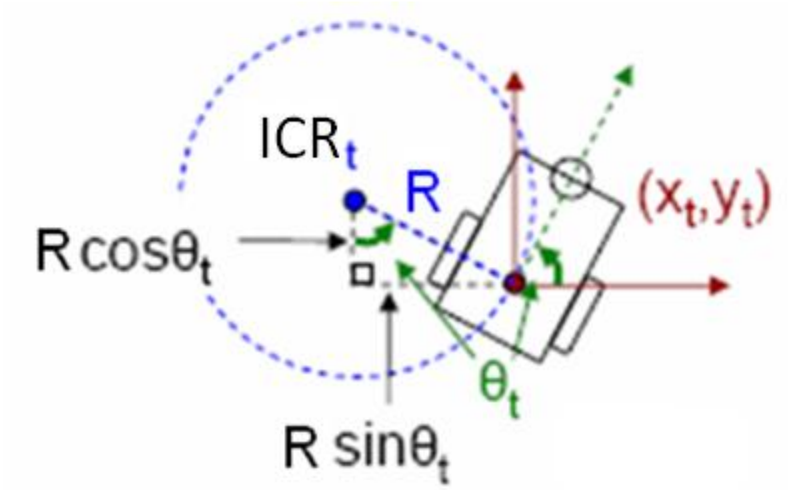
- Given some control parameters (e.g. wheel velocities) determine the pose of the robot
- The position can be determined recursively as a function of the velocity and position,

  $$p_R(t + \Delta) = F(v_1, v_2)\, p_R(t)$$

- To solve for the ICR center at an instance of time use the following

  $$ICR(t) = (ICR_x, ICR_y) =$$
  $$(x_t - R \sin \theta_t, y_t + R \cos \theta_t)$$





Forward Kinematics: Instantaneous Pose

- At time t + Δ, the robot's pose with respect to the ICR is

$$^{G}p_R(t+\Delta) = R(\omega\Delta)^{-1}\,p_R(t) + ICR(t)$$

$$^{G}p_R(t+\Delta) = \begin{bmatrix} ^{G}x_R(t+\Delta) \\ ^{G}y_R(t+\Delta) \\ ^{G}\theta_R(t+\Delta) \end{bmatrix} = \begin{bmatrix} cos(\omega\Delta) & -sin(\omega\Delta) & 0 \\ sin(\omega\Delta) & cos(\omega\Delta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_R(t) \\ y_R(t) \\ \theta_R(t) \end{bmatrix} + \begin{bmatrix} ICR_x \\ ICR_y \\ \omega\Delta \end{bmatrix}$$

$$^{G}p_R(t+\Delta) = \begin{bmatrix} ^{G}x_R(t+\Delta) \\ ^{G}y_R(t+\Delta) \\ ^{G}\theta_R(t+\Delta) \end{bmatrix} = \begin{bmatrix} cos(\omega\Delta) & -sin(\omega\Delta) & 0 \\ sin(\omega\Delta) & cos(\omega\Delta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} R\sin\theta_t \\ -R\cos\theta_t \\ \theta_t \end{bmatrix} + \begin{bmatrix} ICR_x \\ ICR_y \\ \omega\Delta \end{bmatrix}$$

Forward Kinematics: Instantaneous Pose cont.

Since ICR(t) = (ICR_x, ICR_y) = (x(t) – R sin θ, y(t) + R cos θ)

$$
{}^{G}p_R(t+\Delta) = \begin{bmatrix} x(t+\Delta) \\ y(t+\Delta) \\ \theta(t+\Delta) \end{bmatrix} = \begin{bmatrix} \cos(\omega\Delta) & -\sin(\omega\Delta) & 0 \\ \sin(\omega\Delta) & \cos(\omega\Delta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} R\sin\theta_t \\ -R\cos\theta_t \\ \theta_t \end{bmatrix} + \begin{bmatrix} ICR_x \\ ICR_y \\ \omega\Delta \end{bmatrix}
$$

$$
{}^{G}p_R(t+\Delta) = \begin{bmatrix} R\cos(\omega\Delta)\sin\theta_t + R\sin(\omega\Delta)\cos\theta_t + (x_t - R\sin\theta_t) \\ R\sin(\omega\Delta)\sin\theta_t - R\cos(\omega\Delta)\cos\theta_t + (y_t + R\cos\theta_t) \\ \theta_t + \omega\Delta \end{bmatrix}
$$

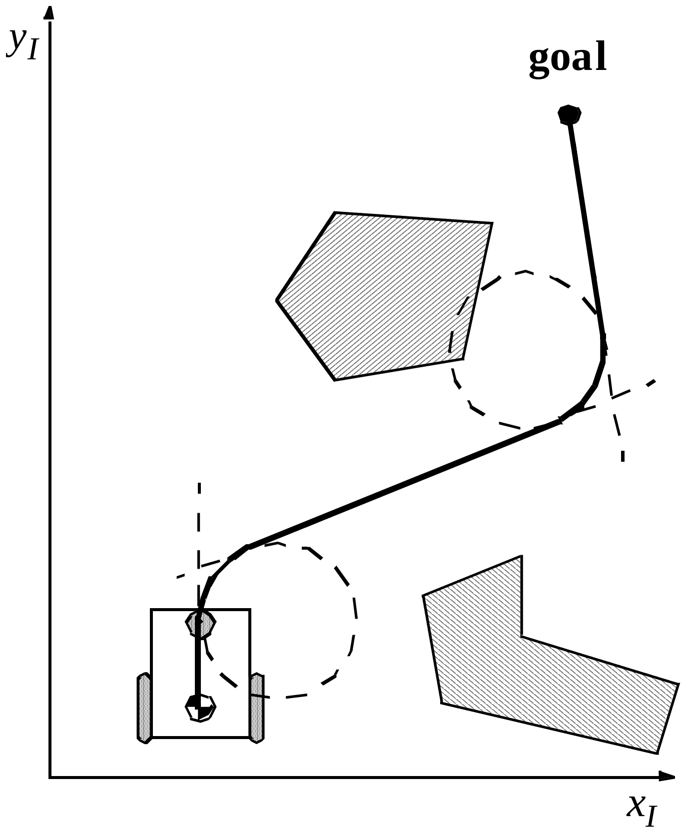
Forward Kinematics: Linear Displacement

- When $v_1 = v_2 = v_t$, $R = \infty$, the robot moves in a straight line so ignore the *ICR* and use the following equations:
- $x(t + \Delta) = x_t + v_t\Delta \cos \theta_t$
- $y(t + \Delta) = y_t + v_t\Delta \sin \theta_t$
- $\theta(t + \Delta) = \theta_t$

Kinematic Controller

- The objective of a ***kinematic controller*** is to have the robot follow a trajectory described by its position and/or velocity profiles as function of time.
- A trajectory is like a path but it has the additional dimension of *time*
- **Motion control (kinematic control)** is not straight forward because mobile robots are non-holonomic systems (and may require the derivative of a position variable).

Kinematic Controller cont.



- One method is to divide the trajectory (path) into motion segments of clearly defined shape:
  - straight lines and segments of a circle. (open loop control)
- control problem:
  - pre-compute a smooth trajectory
    based on line and circle segments

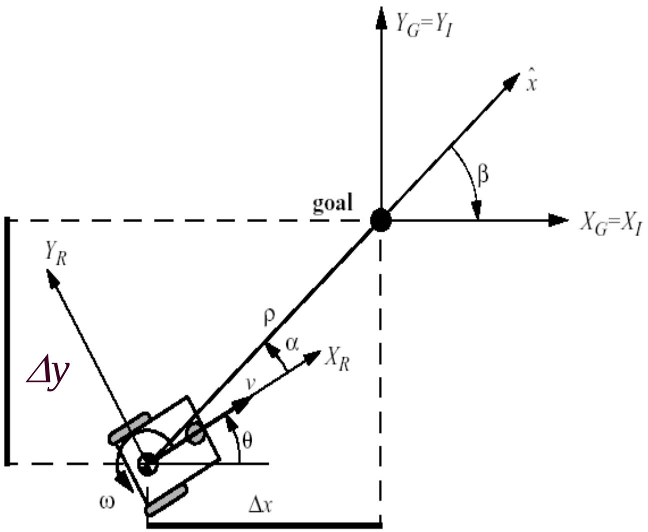
Motion Control
Open-Loop Control

- Disadvantages:
  - It is not easy to pre-compute a feasible trajectory
  - There are limitations and constraints on the robots velocities and accelerations
  - The robot does not adapt or correct the trajectory if dynamic changes in the environment occur.
  - The resulting trajectories are usually not smooth
  - There are discontinuities in the robot’s acceleration
- A more appropriate approach in motion control is to use a real-state feedback controller

Kinematic Model

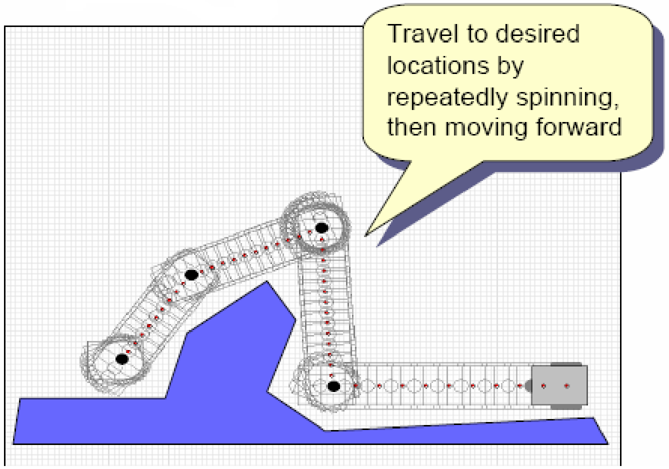
Assume that the goal of the robot is the origin of the global inertial frame. The *kinematics* for the differential drive mobile robot with respect to the global reference frame are:

$$ {}^{G}P_{R} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} $$



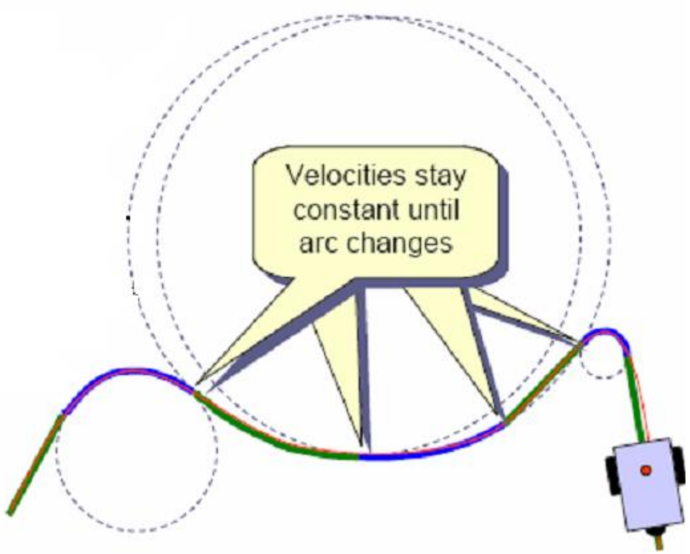
Inverse Kinematics

- *Inverse Kinematics* is determining the control parameters (wheel velocities) that will make the robot move to a new pose from its current pose
- This is a very difficult problem
  - Too many unknowns, not enough equations and multiple solutions
- The easy solution is to
  - Spin the robot to the desired angle
  - Move forward to the desired location



Inverse Kinematics cont.

- Approximate a desired path with arcs based upon computing ICR values
- Result is a set of straight-line paths and ICR arc potions
- Either set the robot drive time and compute velocities for each portion of the path
- Or set velocities and compute drive time for each portion of the path



Inverse Kinematics: Spin time and velocities

- The **spin time** is determined from the wheel velocities
  - $\theta(t + \Delta) = \theta(t) + \omega\Delta \rightarrow \Delta = [\theta(t + \Delta) - \theta(t)]/\omega$
  - Since $\omega = (v_1 - v_2)/(2\ell)$ and $v_1 = -v_2 \rightarrow \omega = v_1/\ell$
  - $\Delta = \ell\,[\theta(t + \Delta) - \theta(t)]/v_1$
- Alternately, set the spin time and calculate the **wheel velocities**
  - $v_1 = \ell\,(\theta(t + \Delta) - \theta(t))/\Delta$

Inverse Kinematics: Forward Time

- The **forward time** is determined by the velocity ($v_t = v_1 = v_2$)
- Since $x(t + \Delta) = x_t + v_t\Delta \cos(\theta_t)$ and $y(t + \Delta) = y_t + v_t\Delta \sin(\theta_t)$
  - if $x(t + \Delta) \neq x_t$
    - $\Delta = (x(t+ \Delta) – x_t)/(v_t\cos(\theta_t))$, or
  - if $x(t + \Delta) = x(t)$
    - $\Delta = (y(t+ \Delta) – y_t)/(v_t\sin(\theta_t))$

Inverse Kinematics: Forward Velocities

- Conversely, the **wheel velocities**, $v_t = v_1 = v_2$, can be determined by setting the forward time
- Since $x(t + \Delta) = x_t + v_t\Delta \cos(\theta_t)$ and $y(t + \Delta) = y_t + v_t\Delta \sin(\theta_t)$
  - if $x(t + \Delta) \neq x_t$
    - $v_t = (y(t+ \Delta) - y_t)/(\Delta \cos (\theta_t))$
  - if $x(t + \Delta) = x(t)$
    - $v_t =(y(t+ \Delta) - y_t)/(\Delta \sin(\theta_t))$