

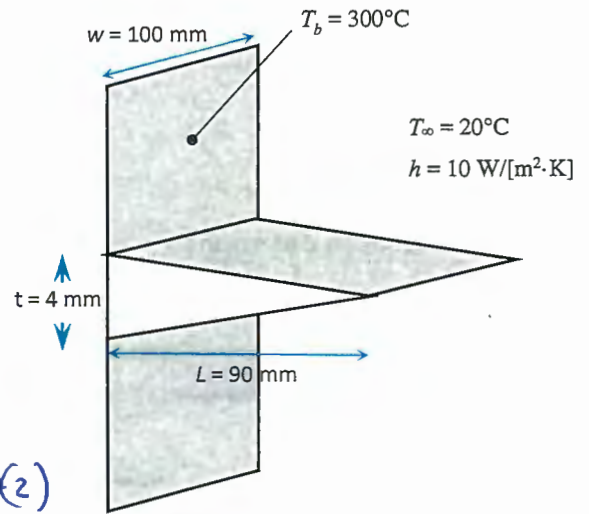
Solutions

Grade: ____/20

Name

Problem 1 (9 pts)

A straight triangular fin made of an aluminum alloy ($k = 50 \text{ W/m}\cdot\text{K}$) is exposed to a convective environment as shown in the figure. The fin is attached to a wall maintained at $T_b = 300^\circ\text{C}$. Dimensions and other information are given in the figure. Ignore heat any heat transfer from the front and back faces of the fin.



- (a) [5 pts] If the fin efficiency is 74%, find the rate of heat transfer from the fin in W.
 (b) [4 pts] Find the fin effectiveness.

$$(a) \dot{Q} = \eta_f \dot{Q}_{MAX} \quad \dot{Q}_{MAX} = h A_{fin} (T_b - T_\infty)$$

$$A_{fin} = \sqrt{L^2 + \left(\frac{t}{2}\right)^2} \cdot w \cdot 2$$

$$= \left(\sqrt{0.090^2 + 0.002^2} \cdot \text{m} \right) (0.100 \text{ m}) (2)$$

$$= 0.018 \text{ m}^2$$

$$\dot{Q} = \left(0.74 \cdot 10 \frac{\text{W}}{\text{m}^2 \cdot \text{K}} \right) (0.018 \text{ m}^2) \cdot (300 - 20)^\circ\text{C}$$

$$= 37.3 \text{ W} \leftarrow$$

ANS

$$(b) \epsilon = \frac{\dot{Q}_{fin}}{\dot{Q}_{no\ fin}} = \frac{\eta_f h A_{fin} (T_b - T_\infty)}{h A_b (T_b - T_\infty)} = \eta_f \frac{A_{fin}}{A_b}$$

$$= \eta_f \frac{A_{fin}}{t \cdot w} = 0.74 \frac{0.018 \text{ m}^2}{(0.004 \text{ m}) (0.100 \text{ m})}$$

$$= 33.3 \leftarrow$$

ANS

Problem 2 [3 pts]

- (a) True | ~~False~~ The fin equation represents conservation of energy for a system consisting of an entire fin.
 (b) ~~True~~ | False The solution to the fin equation can be used to find how temperature changes a long a fin.
 (c) ~~True~~ | False The solution to the fin equation can be used to find the rate of heat transfer from a fin.

Problem 3 (4 pts)

Answer the following questions.

- (a) [1 pt] Insulation of thickness L is added to a plane wall. The *total* thermal resistance of the wall/insulation combination will
- A. decrease
 - B. stay the same
 - C. increase
 - D. Cannot be determined
- (b) [1 pt] Insulation of thickness L is added to the outside diameter of a hot water pipe. The *total* thermal resistance of the pipe/insulation combination will
- A. decrease
 - B. stay the same
 - C. increase
 - D. Cannot be determined
- $r_{NEW} < r_{crit}?$
- (c) [1 pt] The handle of a metal spoon is 7.0 cm long and modeled as a fin. You calculate the parameter m to be $m = 0.879 \text{ cm}^{-1}$. Is the infinitely long finite boundary condition a reasonable assumption?
- A. Yes
 - B. No
 - C. Cannot be determined
- $L_{fin} \gg \frac{1}{m}$
- (d) [1 pt] For the spoon in part (c), is the insulated tip with a corrected fin length boundary condition a reasonable assumption?
- A. Yes
 - B. No
 - C. Cannot be determined
- Always a good choice

Problem 4 (4 pts)

An aluminum ($k=230 \text{ W/m-K}$, $\rho=2600 \text{ kg/m}^3$, $c=901 \text{ J/kg-K}$) cube with side length $L=2.00 \text{ mm}$ is initially at a temperature of $T_1 = 60^\circ\text{C}$. Suddenly it is placed in a convective environment with $T_\infty=20^\circ\text{C}$ and $h_\infty=80 \text{ W/m}^2\text{-K}$ where all faces are exposed to the convective fluid.

- (a) [2 pts] Can you assume that temperature gradients are negligible with the aluminum? Justify your answer *quantitatively*.
- (b) [2 pts] The time constant is calculated to be $TC = 9.8$ seconds. Estimate the temperature of the aluminum 30 minutes after it is exposed to the environment.

(a)
$$Bi = \frac{hL_{char}}{k} = \frac{h \frac{V}{A_{sur}}}{k} = \frac{h \cdot L^3 / (6L^2)}{k} = \frac{hL}{6k} = \frac{80 \frac{\text{W}}{\text{m}^2\cdot\text{K}} \cdot 0.002 \text{ m}}{6 \cdot 230 \frac{\text{W}}{\text{m}\cdot\text{K}}}$$
$$= 0.00116 \ll 0.1$$
 Temperature gradients are negligible.

(b) $30 \text{ min} = 1800 \text{ s} \gg TC$

$$T_{final} \approx T_\infty = 20^\circ\text{C}$$

ANS