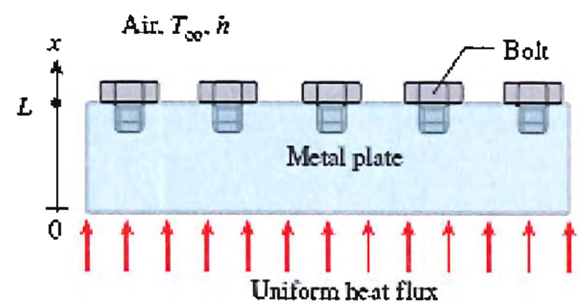
Grade: ______/20

Problem 1 (11 pts)

A metal plate with thickness L and thermal conductivity k is subjected to a constant uniform heat flux of q on its bottom surface. The upper surface of the plate is exposed to ambient air with a temperature and convection heat transfer coefficient of T_∞ and h, respectively. The thickness L is much smaller than the other two dimensions of the plate.

(a) [3 pts] Ignoring any effect the bolts at the top surface may have, what is the simplest form of the conduction equation for the plate for steady conditions?



- A. $\frac{d^2T}{dx^2} + \frac{\dot{e}_{gen}}{k} = 0$
- B. $\frac{d}{dx}\left(k\frac{dT}{dx}\right) = 0$
- C. $\frac{1}{\alpha}\frac{dT}{dt} + \frac{d^2T}{dx^2} = 0$
- D. $\frac{d^2T}{dx^2} = 0$

(b) [1 pt] How many initial conditions would be necessary to solve this differential equation for the temperature distribution?

- A. 0
- B. 1
- C. 2
- D. 4

(c) [2 pts] What would be the *total* number of boundary conditions necessary to solve this differential equation for the temperature distribution?

- A. 1
- B. 2
- C. 3
- D. 4

(d) [2 pts] If one boundary condition is required, give a complete, mathematically correct expression for it. If more than one boundary condition is required, choose one and write a complete, mathematically correct expression for it.

@ $x=0$

$$-k\left.\frac{dT}{dx}\right|_{x=0} = q$$

@ $x=L$

$$-k\left.\frac{dT}{dx}\right|_{x=L} = h\left(T_{x=L} - T_\infty\right)$$

(e) [3 pts] Explain in words how the inclusion of the bolts at the top surface would change the analysis.

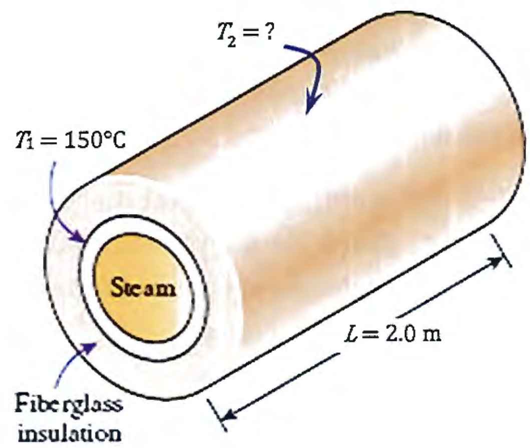
Conduction is now poorly modeled as 1-D in the x-direction. The 2 or 3-D conduction equation would need to be used for both bolts & the plate.

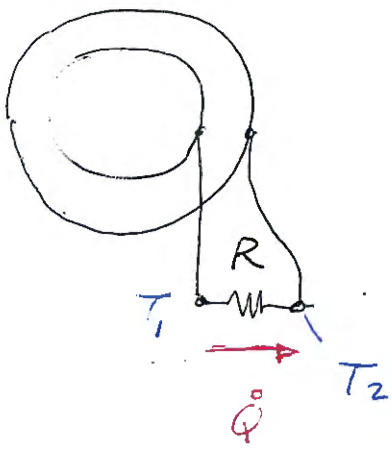
Name [1 pt]

Problem 2 (8 pts)

A 2-m-long section of a steam pipe with outer diameter is $D_1 = 10$ cm and temperature $T_1 = 150°C$ experiences a heat loss of $\dot{Q} = 170$ W. Insulation with thermal conductivity $k = 0.035$ W/m-°C and thickness $t = 1.92$ cm surrounds the pipe.

Assuming steady, one-dimensional conduction, find the outside temperature of the insulation, T_2.





$$\dot{Q} = \frac{T_1 - T_2}{R}$$

$$T_2 = T_1 - \dot{Q}R$$

$$= T_1 - \dot{Q}\frac{\ln(r_o/r_i)}{2\pi k L}$$

$$T_2 = T_1 - \dot{Q}\frac{\ln\left(\frac{D_1}{2} + t\right)}{2\pi k L}$$

$$= 150°C - 170\,\cancel{W} \cdot \frac{\ln\left[\left(\frac{0.10\,m}{2} + 0.0192\,m\right) / \left(\frac{0.10\,m}{2}\right)\right]}{2\pi \times 0.035\,\frac{\cancel{W}}{\cancel{m}\cdot °C} \cdot 2\,\cancel{m}}$$

$$= 24.4°C$$

ANS