COURSE WORKBOOK: Course learning objectives, notes and examples

for

ME302

Heat Transfer

ROSE-HULMAN
INSTITUTE OF TECHNOLOGY

Learning objectives

ME302 learning objectives

After studying the material and doing the associated activities and homework problems students of this course will be able to:

1. ☐ Find course information on webpages
2. ☐ Explain how Heat Transfer as a separate discipline is different than the study of Thermodynamics
3. ☐ Give the "baby" form of the working equation for each **mode of heat transfer**
4. ☐ Explain what each variable in the equations is
5. ☐ Distinguish between T_{amb} (or T_∞) for **convection** and T_{surr} for **radiation**.
6. ☐ Perform an **energy balance** (conservation of energy) *on a surface* subjected to heat transfer
7. ☐ Use a **thermal energy balance** and distinguish it from the more general conservation of energy
8. ☐ Use a **1-D conduction equation** and distinguish it from the more general conservation of energy
9. ☐ Explain the meaning of each term in the above equations
10. ☐ Explain when it is appropriate to use each of the above equations
11. ☐ Define the terms
    - **heat generation**
    - **thermal diffusivity**
12. ☐ Identify the major use of the general, **3-D conduction equation**
13. ☐ Use the conduction equation
    - in different coordinate systems
    - in multiple dimensions
    - with various assumptions (steady-state, constant properties, etc.)
14. ☐ Find **boundary conditions and initial conditions** for use with the conduction equation
15. ☐ Find an expression for Q_{dot} for 1-D, steady-state conduction in rectangular coordinates
16. ☐ Using the electrical/resistance analogy, state what is analogous to V, I, and R.
17. ☐ Express the generic convection relation using an electrical analogy
18. ☐ Draw thermal "circuits" for 1-D, steady-state conduction problems and use them to find unknown temperatures, heat transfer rates, etc.
19. ☐ Find an expression for Q_{dot} for 1-D, steady-state conduction in cylindrical and spherical coordinates

20. ☐ Explain why rectangular coordinate expression for R_{th} does not work in cylindrical and spherical coordinates
21. ☐ Draw thermal "circuits" for 1-D, steady-state conduction problems and use them to find unknown temperatures, heat transfer rates, etc.
22. ☐ Explain why the rectangular coordinate expression for R_{th} does not work in cylindrical and spherical coordinates
23. ☐ Explain what a **fin** is, what it does and how
24. ☐ Explain why the conduction equation and the resistance analogy *cannot* be used to find the temperature distribution in, or heat transfer of, a fin
25. ☐ State how to use **the insulated-tip BC** for a fin to approximate a convective tip
26. ☐ Define **fin efficiency** mathematically and in words
27. ☐ Find the **temperature distribution** for **extended surfaces** (fins)
28. ☐ Find the rate of heat transfer from extended surfaces (fins)
29. ☐ Explain what a **fin effectiveness** is and how it is different from fin efficiency
30. ☐ Use the idea of fin effectiveness to determine when it is a good idea to use a fin or not
31. ☐ Use fin effectiveness in calculations to determine the rate of heat transfer from individual fins and also from **fin arrays**
32. ☐ State the fundamental assumptions of the **lumped capacitance model** for **transient conduction**
33. ☐ Calculate and explain the physical significance of the **time constant** for transient systems for which the lumped capacitance model is valid
34. ☐ Test for the validity of the lumped capacitance model
35. ☐ Calculate and explain the physical significance of the **Biot number**
36. ☐ Recognize when a 1-D, transient conduction model is an appropriate model for a heat transfer system
    - Use the first-term approximation of the infinite-sum solution for 1-D transient conduction to find $T = T([x \text{ or } r], t)$ and $Q([x \text{ or } r], t)$ **Note:** *Q, not* Q_{dot}!)
    - **Infinite plane wall** (slab)
    - **Infinite cylinder**
    - **Sphere**
37. ☐ Determine when the first-term approximation of the infinite-sum solution for the above is valid
38. ☐ Explain the difference between how *Bi* for 1-D transient conduction models and *Bi* for the lumped capacitance model is calculated
39. ☐ State how these solutions can be used for specified *T* BCs instead of convective BC

40. ☐ Distinguish between a 1-D transient conduction model in a slab and a 1-D transient conduction model in a **semi-infinite medium** and recognize when each model is appropriate

41. ☐ Use the solution to 1-D transient conduction in a semi-infinite medium to find $T = T([x \text{ or } r], t)$ and $Q([x \text{ or } r], t)$ (**Note:** *Q, not* Q_{dot}!)

42. ☐ Determine when a 2-D and 3-D transient conduction model is appropriate for a given heat transfer system

43. ☐ Use the solutions to the 1-D transient conduction of an infinite slab, an infinite cylinder, a sphere, and a semi-infinite medium to find the temperature distributions and heat transfers[i] in various 2-D and 3-D transient heat transfer systems using ☐ **superposition**.

44. ☐ Describe mathematically and in words the following terms
    - **no-slip boundary condition**
    - **viscosity**
    - **shear stress** and **skin friction coefficient**
    - **Nusselt number**
    - **velocity boundary layer**
    - **thermal boundary layer**

45. ☐ Explain why convection at a solid-fluid interface is really just conduction, and give a mathematical expression for it

46. ☐ Describe how **Prandtl number** affects the relative thicknesses of momentum (velocity) boundary layers to thermal boundary layers

47. ☐ Identify the appropriate Nusselt correlation to use based on
    - whether a flow is **laminar** or **turbulent**,
    - boundary condition,
    - whether a **local** or **average** values of h is required

48. ☐ Discern between **form drag** and **friction drag**, and identify the major contributor to each

49. ☐ Give the local variation of h (or Nu) with angle for flow around a cylinder or sphere.

50. ☐ Define **external flow** and contrast it with **internal flow**.

51. ☐ Explain the difference between **developing flow** and **fully-developed flow**.

52. ☐ Explain the difference between ***hydrodynamically*** **developing** vs. **fully-developed flow** and *thermally* **developing flow** vs. **fully-developed flow**

53. ☐ Identify whether an internal flow is developing or fully-developed

54. ☐ Sketch how both **friction factor** (f) and **Nusselt number** (Nu) vary in the flow direction for developing flow and fully developed flow.

55. ☐ Define, in words and mathematically, **mean velocity** and **mean (mixing cup) temperature** for internal flow.

56. ☐ Identify the appropriate temperature difference to use for internal flow *based on boundary condition*.
57. ☐ Define **hydraulic diameter** and explain when it is appropriate to use
58. ☐ Identify an appropriate Nusselt correlation to use for a given internal flow situation
59. ☐ Identify the trade-offs of increasing h by increasing flow rate
60. ☐ Calculate the **friction factor**, **pressure drop**, and **pumping power** for flow through a length of pipe
61. ☐ Explain the difference between **forced convection** and **natural convection**
62. ☐ Explain how and why a fluid subject to natural convection moves
63. ☐ Define **volume expansion coefficient** (β), mathematically and in words
64. ☐ Define **Grashof number** and give its physical interpretation
65. ☐ Sketch what velocity and thermal boundary layers look like for natural convection for $Pr>1$ and $Pr<1$.
66. ☐ Explain what type of forces balance each other in natural convection boundary layers for $Pr>1$ and $Pr<1$.
67. ☐ Find the appropriate Nusselt correlation for natural convection based on Ra, boundary condition, geometry and orientation of surface, etc.
68. ☐ Sketch flow patterns for natural convection currents in **enclosures**
69. ☐ State the driving temperature difference to use in the convection relation for natural convection in enclosures
70. ☐ Find the **effective thermal conductivity** for natural convection in enclosures and use it to determine the rate of heat transfer assuming 1-D SS conduction
71. ☐ **Non-dimensionalize** an equation by substituting dimensionless forms of variables into it
72. ☐ Determine when a system subject to **combined forced and natural convection** has negligible natural convection or negligible force convection
73. ☐ Calculate the Nusselt number and heat transfer coefficient for combined natural and forced convection
74. ☐ Explain the ways in which **radiation** heat transfer is different than conduction and convection
75. ☐ Explain how thermal radiation differs from other forms of E-M radiation
76. ☐ Identify the wavelengths of the E-M spectrum for which thermal radiation is the dominant form of radiation
77. ☐ Define a **blackbody**
78. ☐ Define **emissive power** and give its dimensions along with a set of typical units

79. ☐ Define the term **spectral** and **spectral emissive power**

80. ☐ Sketch **spectral blackbody emissive power** as a function of wavelength with temperature shown as a parameter

81. ☐ Find the fraction of emissive power emitted by a blackbody over a specified wavelength range using the **black body radiation function**

82. ☐ Define the terms **spectral emissivity**, **directional emissivity**, **hemispherical emissivity**, **total/total hemispherical emissivity, absorptivity**, **solar absorptivity**, **reflectivity**, **transmissivity, irradiation** and **opaque**

83. ☐ Relate the above mentioned properties to each other

84. ☐ Use **Kirchoff's law** to relate absorptivity and emissivity to each other

85. ☐ Calculate the net radiation from a surface subject to **solar radiation**

86. ☐ Define (mathematically and in words) and the terms **solid angle**, **radiation**, and **view factor**

87. ☐ Find view factors for diffuse surfaces with common geometries and arrangements

88. ☐ Calculate view factors for infinitely long 2-D bodies using the **crossed string method**

89. ☐ Calculate the net rate of radiation heat transfer leaving a black surface as well as the net exchange of radiation heat transfer between black surfaces forming enclosures by making use of **radiation space resistances**

90. ☐ Calculate the net radiation heat transfer from each surface and the net radiation heat transfer between surfaces in an enclosure made up of **diffuse**, **gray** surfaces by making use of **radiation space resistances** and **radiation surface resistances**

91. ☐ Define, mathematically and in words, the terms **radiosity** and **reradiating surface** and give examples of surfaces that behave as reradiating surfaces

92. ☐ Calculate the net radiation heat transfer between two surfaces that have one or more **radiation shields** between them.

93. ☐ Show why the rate the rate of radiation heat transfer between surfaces is diminished by the presence of radiation shields.

94. ☐ Identify the radiation surface properties and their relative values necessary to make a radiation shield effective.

95. ☐ Describe the construction of a **double pipe heat exchanger**, what it does and how.

96. ☐ For a double pipe heat exchanger in both **parallel flow** and **counter flow** configurations
    - Calculate the **overall heat transfer coefficient** for a double pipe heat exchanger
    - Calculate the **log mean temperature difference** for a double pipe heat exchanger
    - Calculate the rate of heat transfer

97. ☐ Use the ***LMTD-F* method** to perform **heat exchanger design problems**.

98. ☐ Use the ***ε-NTU*** method to perform **heat exchanger analysis problems**.

99. ☐ Define **heat exchanger effectiveness**, *ε*

100. ☐ Define **number of transfer units**, ***NTU***

101. ☐ Define **boiling**

102. ☐ Sketch the **boiling curve** and identify the various regions on it

103. ☐ Define **critical heat flux** and explain the concept of **burnout**

104. ☐ Identify appropriate boiling correlations to find heat flux for various regions on the boiling curve

105. ☐ Explain the difference between **dropwise** and **film condensation**, and identify which one is accompanied by larger heat fluxes

106. ☐ Calculate the **Reynolds number** for film condensation

107. ☐ Find appropriate Nusselt relations for film condensation

Note: Terms in **bold** are key concepts or vocabulary words that you should be able to define. This is true whether or not the learning objective is explicitly to define them.

You and Me and Heat Transfer (Makes Three)

So what is heat transfer?

- Defined in Thermodynamics as

- In Heat Transfer as a separate discipline:
  - We are *usually* interested in the ____________ of heat transfer.

  - We are interested in the __________ of energy transfer.
  - We deal with _________________ processes.
  - We will be interested in the _________________ of temperature.

Why should I care?

- Heat transfer processes are encountered in large numbers of engineering systems and other aspects of life. For example:
  - ________________________
  - ________________________
  - ________________________
  - ________________________

What can I expect to get out of this course?

- A working knowledge of heat transfer such that:
  - you can describe physical systems in terms of heat transfer models
  - you can determine heat transfer rate(s) or temperature distributions for existing systems
  - you can determine the size of a system to achieve a specified heat transfer rate or temperature distribution

Details, I want details!

Who is the hottest person in the room?

- There are three modes of heat transfer. Specifically,
  - ________________
  - ________________ (_________________ + *advection*)
  - ________________

________________ and ________________ require mediums.

________________ does not.

Exercises

1. A 2-kg copper bar (not to be confused with the downtown Terre Haute watering hole) is initially at a temperature of T_1 = 25°C. It is then heated at a constant rate for two minutes until the temperature is T_2 = 80°C. If the specific heat of copper is c = 385 J/kg-°C, find the rate of heat transfer into the copper in W.

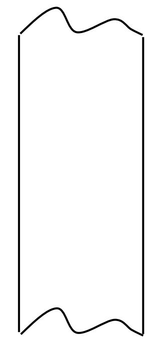
2. The same copper bar is sandwiched between two isothermal walls maintained at constant temperatures. The bar is 15 cm long with a cross sectional area of 2 cm^2. If the hotter of the two walls is 40°C and the thermal conductivity of copper is k = 400 W/m-K, find the temperature of the colder wall for the same rate of heat transfer as in Problem 1.

3. A solid wall is maintained at 50°C. Air at a temperature of 25°C with a convective heat transfer coefficient of 10 W/m^2·°C blows past the wall at a velocity of 0.25 m/s. Find the rate of heat transfer from the wall to the air in W/m^2.

4. The speed of the air blowing past the wall in Problem 3 is increased to 5.0 m/s. Find the new value of the heat transfer coefficient and the new rate of heat transfer.

NOTES: The three modes of heat transfer

Conduction



$\dot{q} =$

Convection

$\dot{q} =$

Radiation

A perfect ________

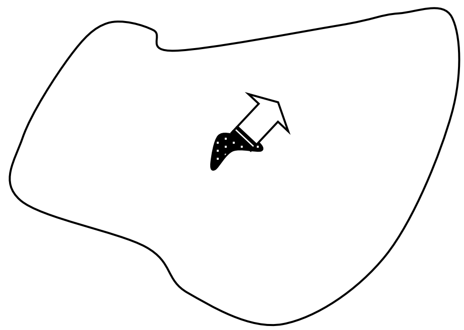
$\dot{q} =$

Not so perfect ________

$\dot{q} =$

Small body enclosed in much larger enclosure

$\dot{q}_{net} =$



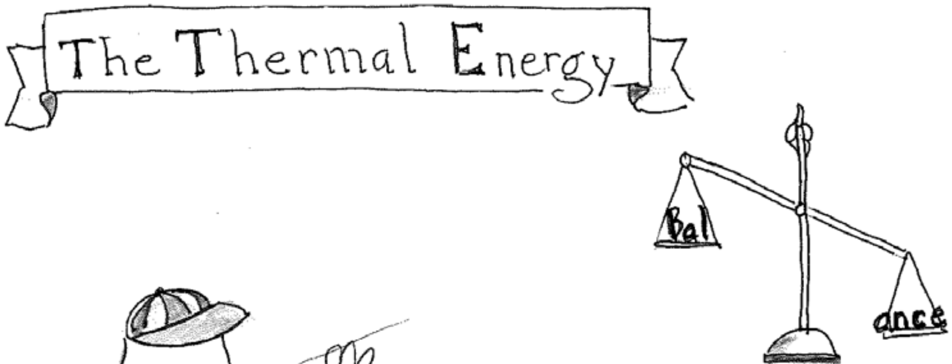
Examples

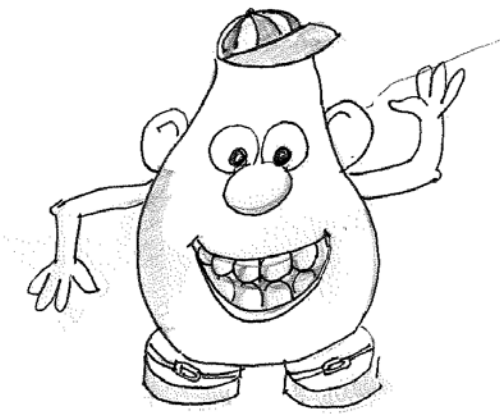
1. A surface area of 2 m^2 has a steady, uniform temperature of $T_{S,out}$ = 13°C and an emissivity of ε = 0.93. The temperature of the surroundings to which this surface radiates is 268 K. Find the net radiation heat transfer (in W) from the surface to the surroundings.

2. Concurrently, air at 10°C blows over the surface. The resulting convective heat transfer coefficient is h = 20 W/m^2-K. Find the convection heat transfer (in W) from the surface to the air.

3. The surface is actually a makeshift roof of a clubhouse. The roof material is 13 mm thick, and the *inside* temperature is $T_{S,in}$ =25°C. Assuming that heat transfer through the roof is one-dimensional and steady, find the thermal conductivity (in W/m·K) of the roof material. (Hint: You will have to make some assumptions about the heat transfer through the roof material to get an answer here. Can you defend your assumptions?)

NOTES: **The thermal energy balance**





Start with a general conservation of energy

Make it a closed system. Do *not* ignore forms of energy besides U and do *not* ignore electrical power. (You can ignore KE & PE and other forms of power, though.)

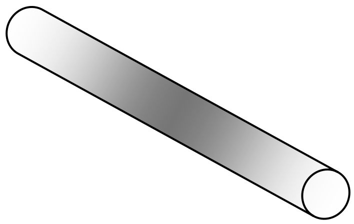
Seeing as how this course is *Heat Transfer*, let's lump everything that is not U together:

$$\boxed{\dot{E}_{gen} \equiv \qquad\qquad\qquad\qquad\qquad\qquad}$$

Put it all together...

Example

A long cylinder of cross section A is insulated along its outer diameter and is subject to a uniform internal heat generation per unit volume of $\dot{e}_{gen}$. Assuming constant conductivity k and specific heat c, find a differential equation describing the temperature distribution as a function of length and time.



Example

The temperature distribution in a wall 1m thick at a certain instant of time is given as

$$T(x) = a + bx + cx^2$$

where T is in °C and x is in m. The constants are a = 900°C, b = -300°C/m and c = - 50°C/m². A uniform heat generation $\dot{e}_{gen}$ = 1000 W/m³ exists in the wall. The wall area is 10 m² and has the following properties: ρ = 1600 kg/m³, k = 40 W/m-K and c_p = 4 kJ/kg-K. Determine:

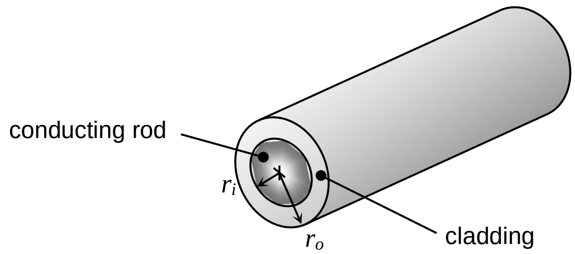
1. the rate of heat transfer entering the wall and leaving the wall. (x=0 and 1 m, respectively),

2. the rate of change of energy storage in the wall, and

3. the time rate of temperature change at x = 0 and 0.25 m.

Example

Electric current is passed through a long conducting rod of radius r_i and thermal conductivity k_r, resulting in a uniform volumetric heat generation of $\dot{e}_{gen}$. The rod is wrapped in an *electrically* non-conducting cladding with outer radius r_o and thermal conductivity k_c. The entire rod/cladding combination is immersed in a flowing fluid with known heat transfer coefficient h and temperature T.



(a) Reduce the conduction equation for steady-state conditions and state the appropriate boundary conditions for the conducting rod.

(b) Reduce the conduction equation for steady-state conditions and state the appropriate boundary conditions for the cladding.

Example

Jeff Spicoli is trying out a new surfboard designed for use on the northern California coast. Since the NoCal waters are noticeably colder than those at Sunset Cliffs, the new board makes use of electrical resistance heating. The surfboard has rectangular cross section and has a width W that is much greater than its thickness H. The bottom of the surfboard is initially in contact with the ocean at its lower surface, and the temperature throughout the board is approximately equal to that of the ocean T_0. Suddenly Spicoli turns on the heater and catches a tasty wave such that an electric current is passed through the entire board and an air-stream of temperature T_∞ is passed over the top surface at a constant rate. The bottom surface continues to be maintained at T_0.

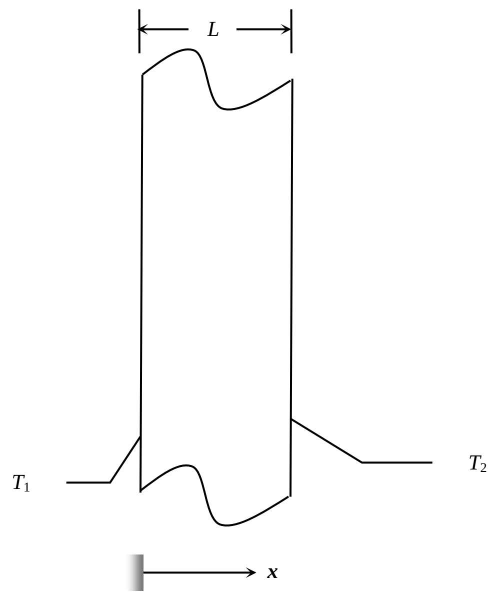
Assuming the board has a constant thermal conductivity k, obtain the differential equation and the boundary and initial conditions that could be used to determine the temperature as a function of time and position in the board.

ACTIVE LEARNING EXERCISE—Thermal resistance

Consider a chunk of material with thickness L and surface area A as shown in the figure. The left hand face is maintained at a constant temperature T_1 while the right hand side is maintained at a constant temperature of T_2. Is the material has a constant thermal conductivity and is subject to 1-D steady-state conduction with no heat generation,

(a) find the temperature distribution $T = T(x)$.
(b) Use your answer to (a) to find an expression for the rate of heat transfer through the chunk, $\dot{Q}$.
(c) Rearrange your answer in (b) to look like

$$\dot{Q} = \frac{T_1 - T_2}{\text{something}}$$

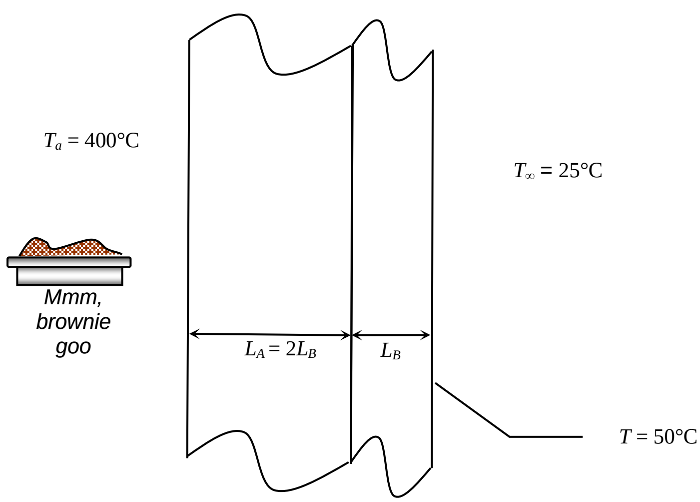


Example

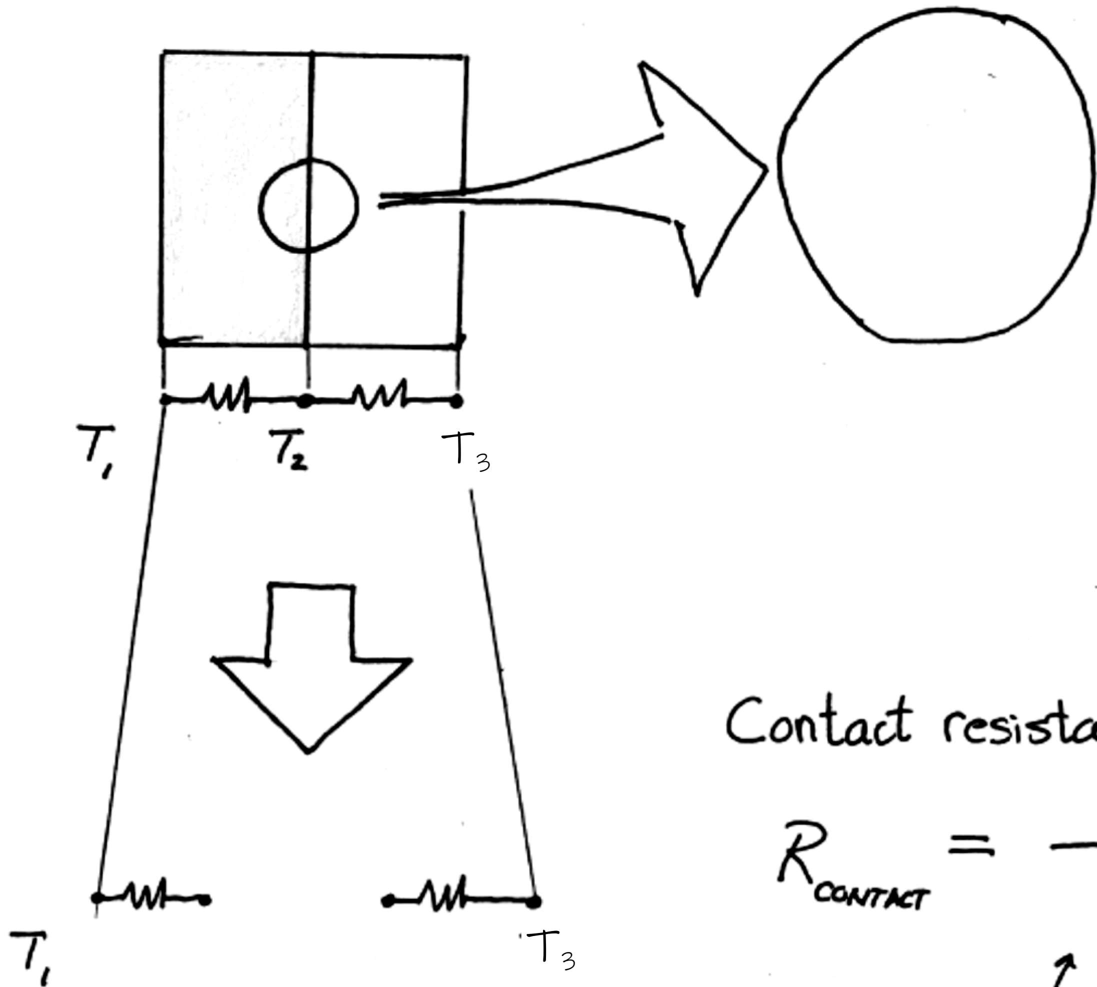
Dr. Thom bakes lots of brownies. In the process, he drips large amounts of brownie goo in his oven. He therefore is looking for a self-cleaning oven. One such oven design involves the use of a composite window separating the oven cavity from the room. The composite consists of two high temperature plastics (*A* and *B*) with thermal conductivities k_A = 0.15 W/(m·°C) and k_B = 0.08 W/(m·K) and thicknesses $L_A = 2L_B$. During the self-cleaning process, the oven air temperature is T_a = 400°C, while the room air temperature is T_∞ = 25°C. Convective heat transfer coefficients in and out of the oven are approximately 25 W/(m^2·°C).

(a) Find the minimum window thickness $L = L_A + L_B$ needed to ensure a temperature of 50°C on the outer window surface. (Hint: Use the resistance analogy and draw a thermal circuit. Assume that the cross sectional area of the window in 1 m^2 to make life easier.)

(b) Repeat part (a) if there is also a *radiation heat transfer coefficient* inside the oven of h_r = 25 W/(m^2·°C).



Contact resistance



Contact resistance:

$R_{CONTACT} =$ ______

Contact ________ ________

$= f($ $)$

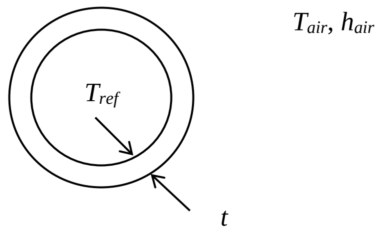
NOTES: Contact resistance

Pick the engineer:

Example

A 10-mm diameter pipe containing a condensing refrigerant is to be insulated with a material that has a conductivity of k_{insul} = 0.055 W/m-°C. For the air surrounding the pipe, T_{air} = 20°C and h_{air} = 5 W/m^2-°C. The temperature of the refrigerant is –10°C. Assuming that the inside wall temperature is the same as the refrigerant temperature

(a) calculate the rate of heat transfer per unit pipe length for an insulation thickness of t = 2 mm, and
(b) t = 5 mm.



NOTES: Fins

WAYS TO INCREASE

$$\dot{Q}_{conv} = h A (T_s - T_\infty)$$

1) INCREASE ________________

+   −

-
-
-

2) INCREASE ________________

+   −

-
-
-

3) INCREASE ________________

+   −

-
-
-

NOTES: Fins

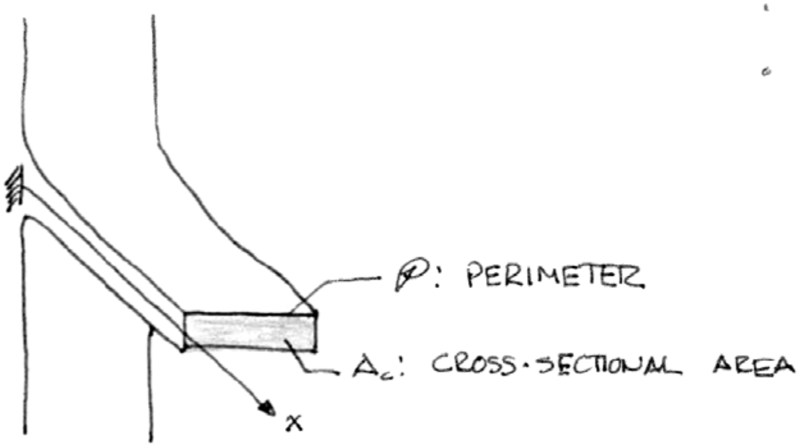
FINS

MODEL A FIN TO GET

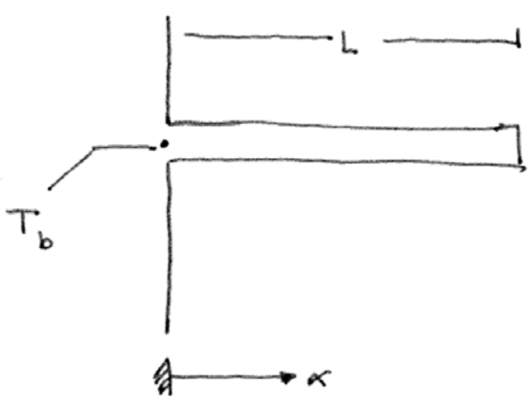
- $T = T(x)$ &
- $\dot{Q}_{FIN} = ?$

ASSUME:

- 1-D CONDUCTION
-
-
-



SIDE VIEW:



Thermal Energy Balance: *

$$\frac{dU}{dt} =$$

* why not use the conduction equation?

NOTES: Fins

$$\frac{d\dot{Q}}{dx} + hP(T_x - T_\infty) = 0$$

WHAT'S $\dot{Q} = ?$

$= - \quad \_\_\_$

$$- \quad \_\_\_ \quad + hP(T_x - T_\infty) = 0$$

LET: $\theta \equiv T_x - T_\infty$

$$\boxed{\_\_\_ \quad - \quad \_\_\_ \quad = 0}$$

FIN EQN (CONST X-SEC AREA)

SOLVE IT! CHR. EQN IS

SO:

WHAT ARE THE BCs?

	IN T	IN $\theta = (T - T_\infty)$
BC#1	$x = 0$	$x = 0$
BC#2	$x = L$	

CHOICES

1)

2)

3)

NOTES: Fins

1) INFINITELY LONG FIN

$T(x=L) =$ $\Theta(x=L) =$ AS $L \to \infty$

SO: (BC #2)

$\theta(x=L \to \infty) =$

$\therefore C_1 =$

BC # 1:

$\theta(x=0) =$

$\therefore C_2 =$

$$\theta = \theta_b e^{-ax} = (T_b - T_\infty) \exp\left[-\sqrt{\frac{hP}{kA_c}}\,x\right]$$

$$\boxed{\frac{T(x) - T_\infty}{T_b - T_\infty} = e^{-\sqrt{\frac{hP}{kA_c}}\,x}}$$

- INFINITELY LONG FIN
- CONST A_c

2) INSULATED TIP

BC # 2

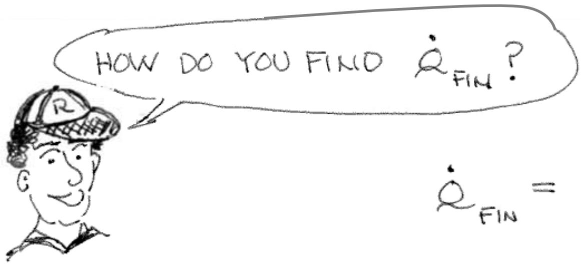
$- \quad \underline{\qquad} =$

$$\boxed{\frac{T_x - T_\infty}{T_b - T_\infty} = \underline{\qquad\qquad}}$$

3) CONVECTIVE TIP

APPROXIMATION

$$\boxed{L_c \triangleq L + \underline{\qquad}}$$

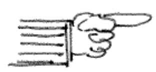


$$\dot{Q}_{FIN} = \qquad \text{———} \Big|_{x=}$$

OOLY LONG ⟹ $\dot{Q} = \sqrt{\qquad\qquad} \; (\qquad\qquad)$

INSUL. TIP ⟹ $\dot{Q} = \sqrt{\qquad\qquad} \; (\qquad\qquad)$

¿WHAT'S THE **BIGGEST** $\dot{Q}_{FIN}$ YOU CAN IMAGINE?



$\dot{Q}_{MAX} =$ ↑________________________ $\neq$

FIN EFFICIENCY

$\eta_f \equiv$ ________

OOLY LONG:

$\eta_f =$ ________

$$\eta_{f,\infty} = \text{—} \sqrt{\qquad\qquad}$$

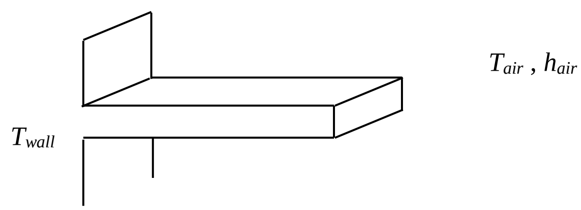
$$\eta_{f,INS} = \qquad \text{————}$$

OTHER GEOMETRIES VIA CHARTS IN YOUR TEXT OR OTHERS.

Example

A straight aluminum fin (k = 200 W/m-K) is 3.00 mm thick and 7.5 cm long. It protrudes from a wall whose temperature is maintained at 300°C. The ambient air temperature is T_{air} = 50°C with h_{air} = 10 W/m^2-K. Calculate the heat loss from the fin per unit depth assuming

(a) an infinitely long fin, and
(b) an insulated tip with a corrected fin length.

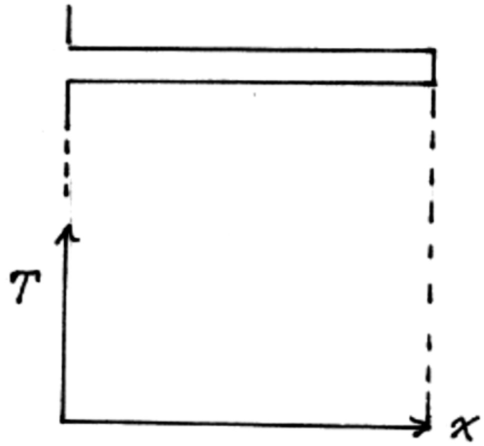


Example

(c) Repeat part b) using the fin efficiency concept.

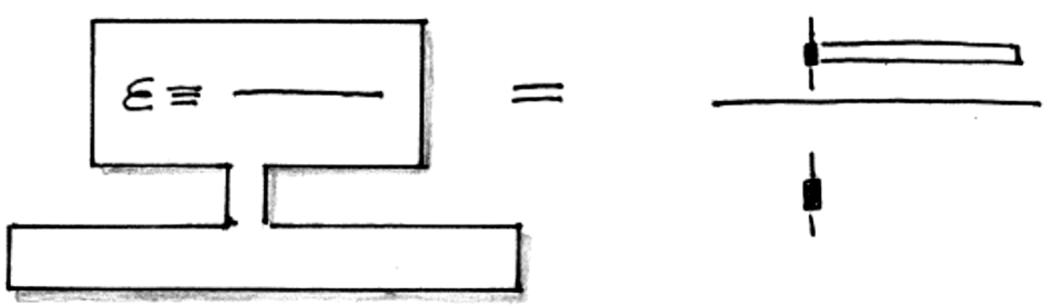
NOTES: Fin effectiveness

Recall fin efficiency



$$\eta = \frac{\dot{Q}_{FIN}}{\dot{Q}_{MAX}}$$

Fin Effectiveness



Limits on ε?

What *should* ε be?

Let's relate ε to η

$$\varepsilon = \frac{\dot{Q}_{fin}}{\dot{Q}_{no\,fin}} = \frac{\eta_f}{\qquad\qquad\qquad}$$

$$\boxed{\varepsilon = \quad \frac{\quad}{\quad}}$$

For an infinitely long straight fin

$$\varepsilon = \left(\qquad\qquad \right) \frac{A_{fin}}{A_{no\,fin}}$$

$A_{fin} =$

$A_{no\,fin} =$

$\varepsilon =$

So, you should use a fin when

k is HIGH | LOW $\longrightarrow$

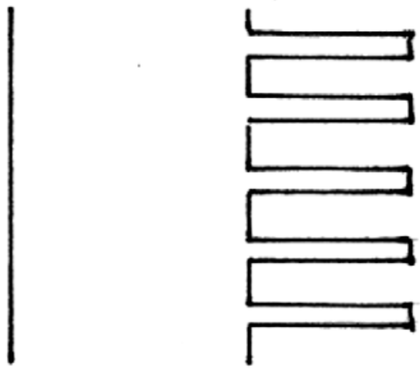
P/A_c is HIGH | LOW $\longrightarrow$

h is HIGH | LOW $\longrightarrow$

All of this is for a single fin...

finder

Fin arrays



$\varepsilon_{OVERALL} =$ ______________

$\dot{Q}_{TOT} = \dot{Q} + \dot{Q}$

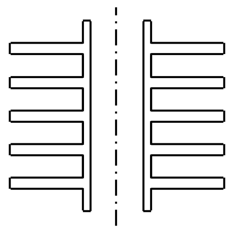
$=$

$\dot{Q}_{NO\ FIN} =$

$\boxed{\varepsilon_{OVERALL} = \text{______________}}$

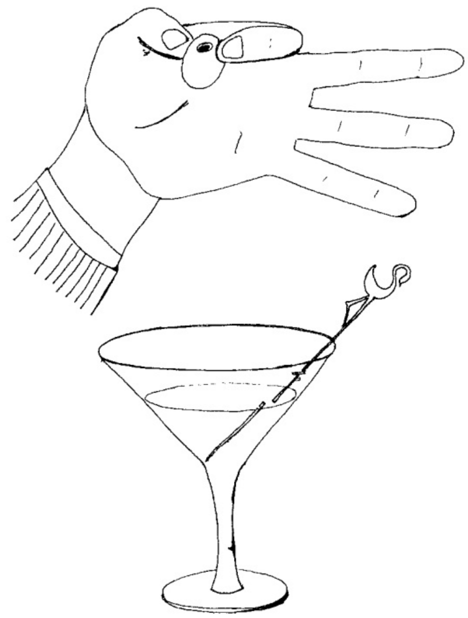
Example

A motorcycle *cylinder* is constructed from 2024-T6 aluminum alloy (k = 186 W/m-°C) and has a height of H = 0.15 m and an outer diameter of D = 50 mm. The temperature of the outer diameter of the cylinder is 500 K under typical conditions. The surrounding air has a temperature is T_{air} = 300 K with h_{air} = 50 W/m^2-K. It is suggested that the heat transfer from the motorcycle can be enhanced by adding *annular* fins of length L = 20 mm and thickness t = 6 mm. Calculate the increase of heat transfer due to adding five such fins, all equally spaced.



ACTIVE LEARNING EXERCISE: The lumped capacitance method

Consider a frozen olive initially at a temperature of T_i that is dropped into a martini at a temperature T_∞. We then stir the martini with a flamingo swizzle stick. We are interested in how the olive temperature changes with time, most notably how long it takes to warm up to T_∞.



Write **thermal energy balance** for the frozen olive for the time after is dropped into the martini. *Assume that the entire olive is at only one temperature at any point in time*. This is the **lumped capacitance assumption**.

What is the mode of heat transfer to the olive? _________________.

Rewrite the thermal energy balance.

This is a linear, non-homogeneous first order differential equation. We can make is homogeneous by letting

$$\theta = T - T_\infty$$

Do it!

Solve by direct integration:

Apply the initial condition:

The solution to this equation is given by

Rearrange a bit

$$\frac{T - T_\infty}{T_i - T_\infty} =$$

where

$$TC =$$

Now this model says that the olive never reaches T_∞, but it is generally accepted that 4τ is close enough. (At $4 \cdot TC$ you're 98% of the way there).

If the convective heat transfer coefficient between an olive and the martini is h = 100 W/(m$^2 \cdot$K) and the properties of a typical 2-cm diameter spherical olive are given by ρ = 850 kg/m^3 and c_p = 1780 J/(kg$\cdot$K), we can calculate TC to be

$$TC =$$

which means that in about _________________ (or $4 \cdot TC$) the olive has reached T_∞.

In this, we *assumed* that the entire olive was at one temperature. In other words, we ignored any temperature gradients within the olive and therefore any _________________ heat transfer within it.[1] Was this a good assumption? Let's find out.

The _________________ _________________ is a measure of the internal resistance to conduction of an object to the external convection to which it is subject. It is defined as

$$Bi \equiv \underline{\hspace{8cm}} = \underline{\hspace{5cm}}$$

[1] Actually, we're not ignoring it as much as we are assuming that it is infinitely efficient!

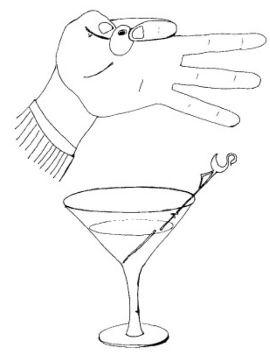
If the Biot number is small (Bi << 1) then this assumption isn't too bad. With k_{olive} = 0.350 W/(m² ·C°) and $L_{char} = V_0/A = r/3$, for the macro-olive we get

$$Bi = \frac{100 \dfrac{\mathrm{W}}{\mathrm{m}^2 \cdot \mathrm{C}^{\mathrm{o}}} \cdot (0.01/3)\ \mathrm{m}}{0.350 \dfrac{\mathrm{W}}{\mathrm{m} \cdot \mathrm{C}^{\mathrm{o}}}} =$$

$Bi << 1$	$Bi = 1$	$Bi >> 1$

Example

Let's take one last look at the frozen olive problem. We drop a frozen olive initially at a temperature of T_i = 0°C into a martini at a temperature T_∞ = 5°C. We then stir the martini with a flamingo swizzle stick resulting in a convection coefficient of h = 10 W/(m² ·C°). The olive is modeled as a sphere with 1-cm diameter with ρ = 850 kg/m³, k = 0.350 W/(m² ·C°) and c_p = 1780 J/(kg ·C°)

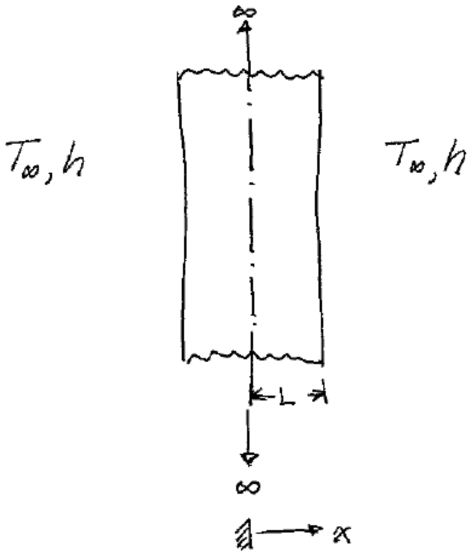


(a) Find the Biot number for the olive in the martini. Is the lumped capacitance model OK?
(b) Find the time constant for the olive in the martini.
(c) How long does it take the olive to warm up to 4°C?
(d) What it the *rate* of heat transfer into the olive when T = 4°C? What is the total amount of heat transferred (Q with no dot!) to the olive during this time?

NOTES: Transient conduction

TRANSIENT 1-D CONDUCTION

- TAKE A SLAB ________ __ ________
- PUT IT IN A ________ ________



CONDUCTION EQN

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{e}_{gen}$$

B.C. # 1: @ $x = L$

B.C. # 2: @ $x = -L$

I.C. $T(x, t=0) =$

SOLVE BY SEPARATION of VARIABLES. USE ________ ________ FOR CONSTANTS (FROM B.C.s)

RESULT IN ________ ________

$$\boxed{\theta =}$$

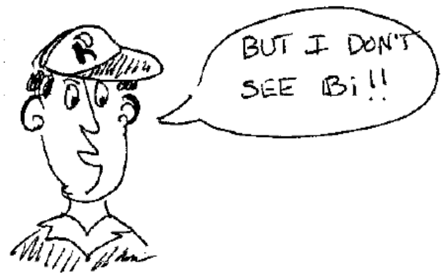
1ST TERM APPROX of ∞ SERIES

$\theta(x,t) \equiv$ ________

WHERE

$\mathrm{Fo} =$ ______

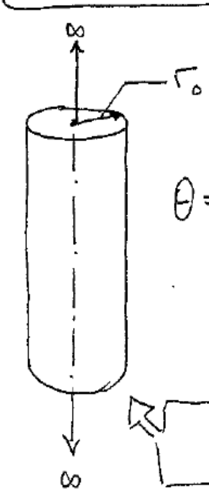
$\mathrm{Bi} =$ ______



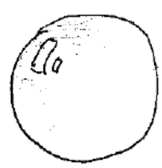
$A_1 = f(\quad)$

$\lambda_1 = f(\quad)$

OTHER GEOMETRIES



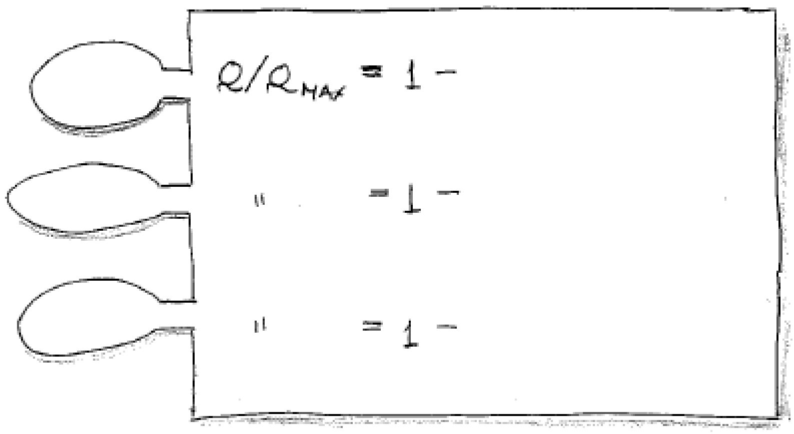
$$\theta = A_1 e^{-\lambda_1^2 \mathrm{Fo}} J_0\left(\frac{\lambda r}{r_0}\right)$$



$$\theta = A_1 e^{-\lambda^2 \mathrm{Fo}} \left(\frac{\sin\left(\lambda_1 \frac{r}{r_0}\right)}{\lambda_1 \frac{r}{r_0}} \right)$$

Q ___ ___ !!

SINCE THIS IS ____________ & WE ARE LOOKING AT ________ ________ LET'S FIND Q (   ) INSTEAD.



WHERE

$\theta_o = \theta$ AT CENTER POINT =

AND

$Q_{MAX} =$

NOTES: Transient conduction

CONCEPT QUESTIONS - Transient conduction

1. For the following questions, assume that the conductive body in question is initially all at one temperature, T_i and is put into a convective environment at time $t = 0$. The convective environment has a heat transfer coefficient of h and is at temperature T_∞.

   a. Find an expression for the dimensions temperature (θ) at the center of an infinite slab of half thickness L as a function of time.

   b. Find an expression for the dimensions temperature (θ) at the center of an infinitely long cylinder as a function of time.

   c. Find an expression for the dimensions temperature (θ) at the center of a solid sphere as a function of time.

   d. Comment on your answers to a-c.

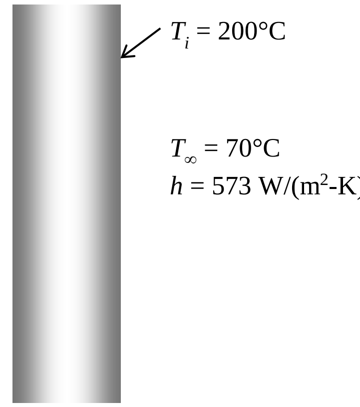
2. Find an expression for the *maximum* heat that can be transferred (Q with no dot) to a slab, infinitely long cylinder or sphere as described in problem 1. (Hints: At what *time* does Q_{max} occur? What is the temperature of *the entire body* at this time?)

Example

A one meter long aluminum cylinder 15.0 cm in diameter and initially at 200°C is suddenly exposed to a convection environment at 70°C and h = 573 W/(m^2-K).

(a) Calculate the temperature at a radius of 1.73 cm 1 min after the cylinder is exposed to the environment.

(b) Calculate the heat lost 1 min after the cylinder is exposed to the environment. Express your answer in J.

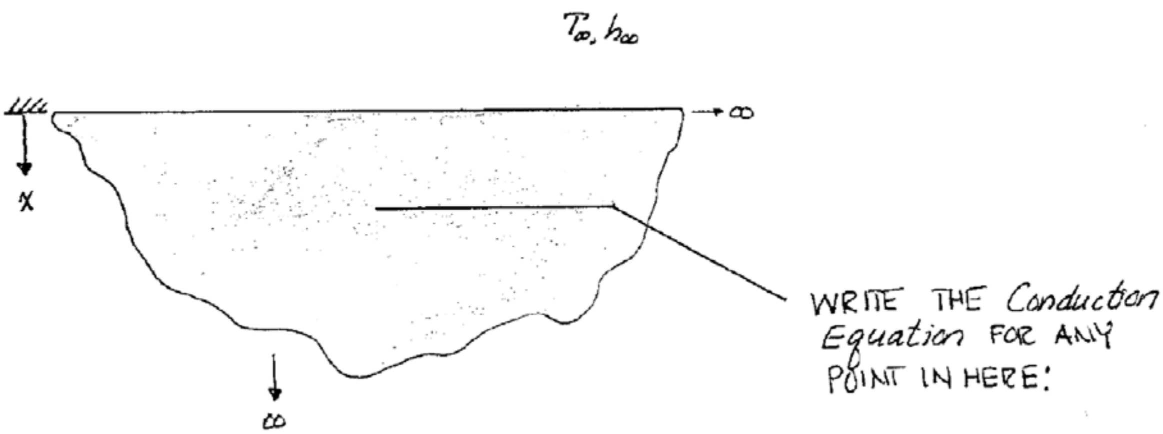


NOTES: Conduction in a semi-∞ medium

CONDUCTION IN A SEMI-∞ MEDIUM (TRANSIENT, THAT IS.)

A SEMI-∞ MEDIUM, INITIALLY AT T_i THROUGHOUT IS SUDDENLY EXPOSED TO A CONVECTIVE MEDIUM WITH h & T_∞.

FIND: $T = T(x, t)$



REDUCE IT (PER ASSUMPTIONS)

[EQN 1]

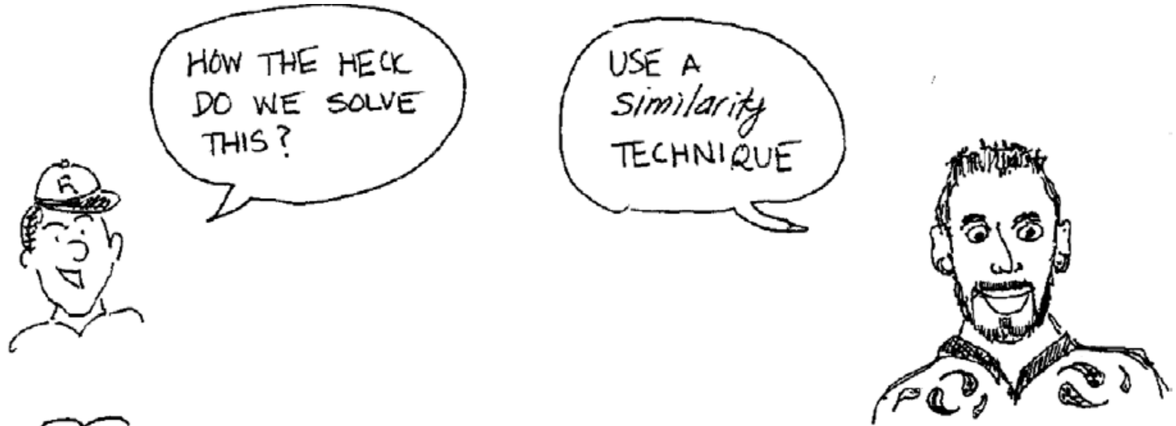
INITIAL & BOUNDARY CONDITIONS

I.C.

B.C. # 1

B.C. # 2

NOTES: Conduction in a semi-∞ medium



DEFINE:

$$\eta \equiv \frac{x}{(4\alpha t)^{1/2}}$$

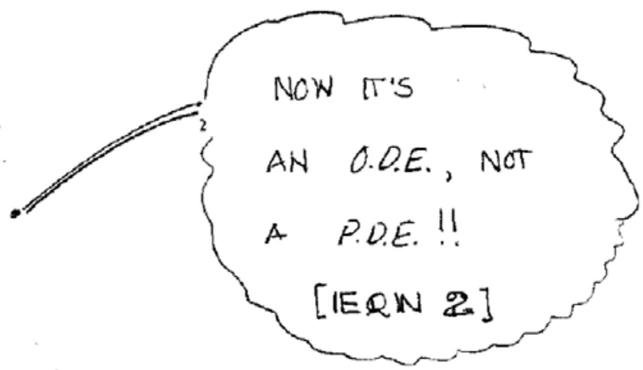
SIMILARITY VARIABLE

TRANSFORM DERIVATIVES:

$$\frac{\partial T}{\partial t} = \frac{dT}{d\eta} \times \frac{\partial \eta}{\partial t} = \Big[\qquad\qquad\qquad \Big] \frac{dT}{d\eta}$$

$$\frac{\partial^2 T}{\partial x^2} =$$

SUBSTITUTE INTO [1]



TRANSFORM I.C. & B.C.s

- I.C. & B.C. #1 COLLAPSE INTO 1 B.C. *(If you can't make this happen, you can't use a similarity technique...)*

$$\left.\begin{aligned} T(x, t=0) &= T_i \\ T(x \to \infty, t) &= T_i \end{aligned}\right\} \quad T(\qquad) =$$

B.C. #2

$$+k \frac{\partial T}{\partial x}\bigg|_{x=0} = h(T_{x=0} - T_\infty)$$

NOW LET'S INTEGRATE [2]

$$\frac{1}{dT/d\eta}\, d\left(\frac{dT}{d\eta}\right) = -2\eta$$

$dT =$

$T =$

$T =$

C_1 & C_2 COME FROM ________________

BLAH, BLAH, BLAH...

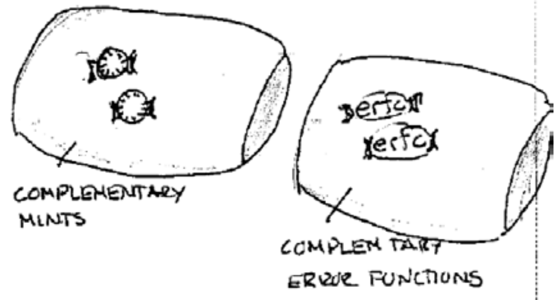
RESULTS:

$$\boxed{\frac{T(x,t) - T_i}{T_\infty - T_i} = \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) - \exp\left(\frac{hx}{k} + \frac{h^2 \alpha t}{k^2}\right) * \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}} + \frac{h\sqrt{\alpha t}}{k}\right)}$$

where

$$\operatorname{erfc}(u) \equiv 1 - \frac{2}{\sqrt{\pi}} \int_0^u \exp(-u^2)\, du$$

COMPLEMENTARY ERROR FUNCTION



NOTE

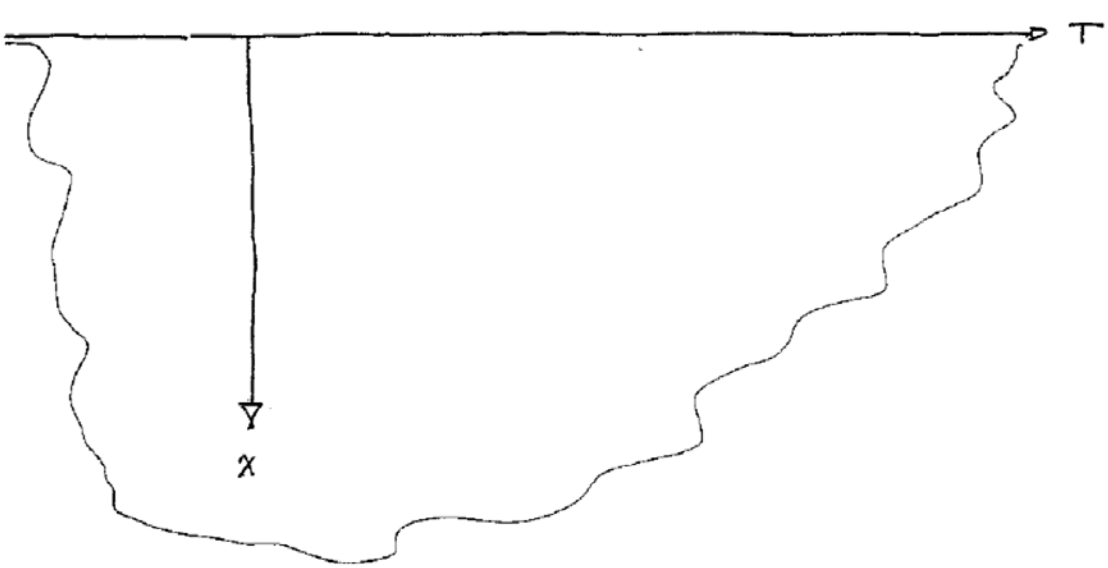
$$\frac{T(x,t) - T_i}{T_\infty - T_i} = 1 - \theta$$

PER OUR PREVIOUS NOTATION.

IF B.C. #2 IS $T(x=0,t) = T_s$ INSTEAD:

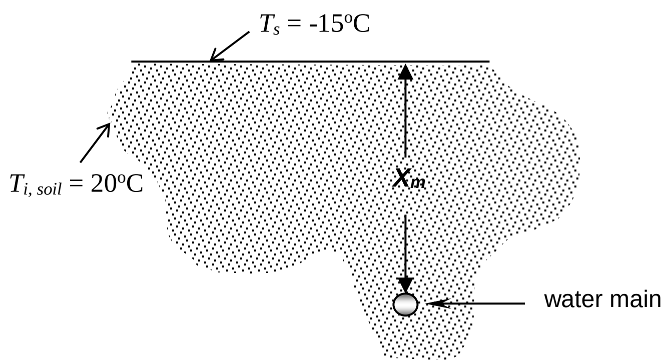
$$\boxed{\frac{T(x,t) - T_i}{T_\infty - T_i} = \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right)}$$

WHAT DO YOU THINK $T(x,t)$ LOOKS LIKE?



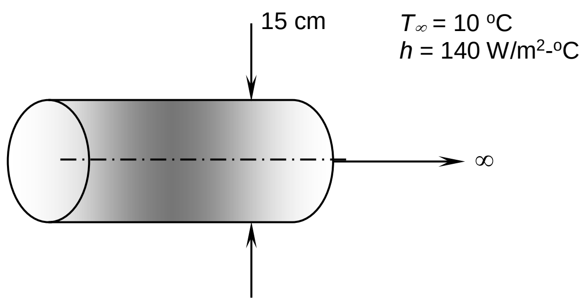
Example

In laying water mains, utilities are concerned about the possibility of freezing during cold periods. What minimum burial depth would you recommend for a water main under the following conditions: Soil, initially at a uniform temperature of 20°C, is subjected to a constant surface temperature of –15°C for 60 days. Assume the properties of soil to be $\rho = 2050$ kg/m^3, $k = 0.52$ W/m-°C, $c = 1840$ J/kg-°C and $\alpha = (k/\rho c) = 0.138 \times 10^{-6}$ m^2/s.



Example

A semi-infinite aluminum cylinder (k = 237 W/m-°C, α = 9.71 x 10^{-5} m^2/s) of diameter D = 15 cm is initially at a uniform temperature of T_i = 150°C. The cylinder is now placed in water at 10°C, where the convection heat transfer coefficient is h = 140 W/m^2-°C. Determine the temperature at the center of the cylinder 10 cm from the end surface 8 min after the start of the cooling.

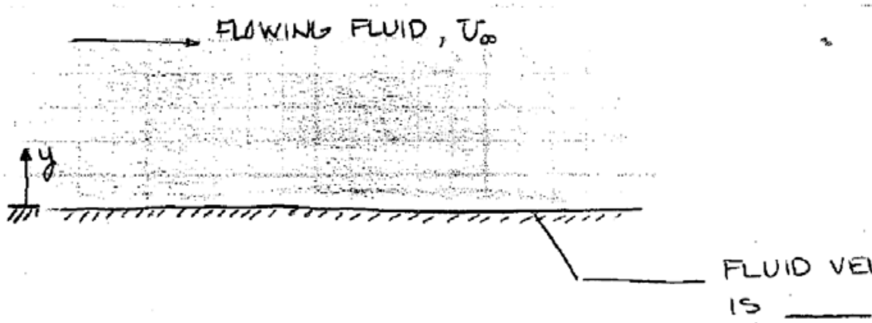


CONVECTION

CONVECTION INVOLVES $\dot{Q}$ BETWEEN A SOLID SURFACE AND

A ____________ ____________

IT ∴ BEHOOVES US TO REVIEW FLUIDS (A LITTLE)



TWO KINDS of STRESS σ IN A FLUID

$\frac{F}{A_{NORMAL}} \Rightarrow$ (USUALLY THIS IS ____________)

$\frac{F}{A_{TANGENT}} \Rightarrow$ $= \boxed{\tau}$

IN A *Newtonian Fluid*

$\tau = \text{——}$

LOOK FAMILIAR? IT SHOULD!! WHAT DOES THIS REMIND YOU OF?

ANYWAY, THE FLOWING FLUID EXERTS A *DRAG FORCE* ON THE SURFACE. WE'D LIKE TO KNOW WHAT THAT IS

NOTES: **Intro to convection**

IN TERMS of τ

$F_D =$ ________ JUST LIKE

IN HEAT TRANSFER

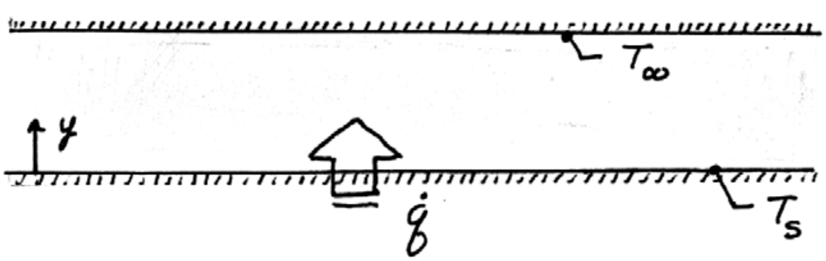
IN *DIMENSIONLESS* FORM

$$\boxed{C_f \equiv \_\_\_\_\_\_\_\_}$$

FRICTION COEFFICIENT

NOW LET'S FOCUS ON THE HEAT TRANSFER

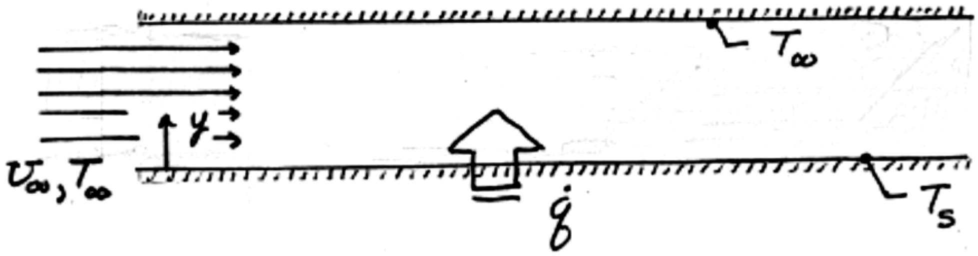
FIRST, LOOK AT A ________ FLUID BETWEEN TWO PLATES



- WHAT IS THE *MODE* OF HEAT TRANSFER? ________

$\therefore\ \dot{q} =$ ______ $=$ ________

NOW LET'S MOVE THE FLUID



• WHAT IS THE MODE NOW? ____________

$\therefore \dot{q} =$ ___

WHY?

GIVES US A WAY TO FIND h ANALYTICALLY:

$$\boxed{h = \rule{4cm}{0.4pt}}$$

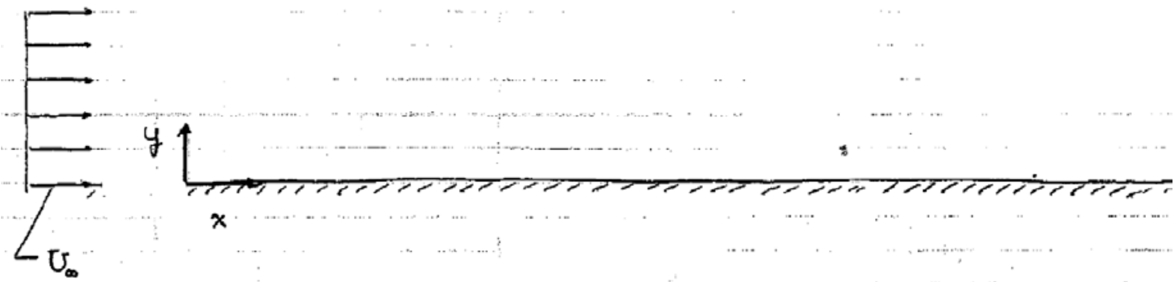
MAKE IT DIMENSIONLESS:

$$\boxed{Nu = \frac{h}{\rule{2cm}{0pt}}}$$

NOTES: Intro to convection

BOUNDARY LAYERS

VELOCITY (MOMENTUM) B.L. →



- INSIDE THE B.L. ______________
- OUTSIDE THE B.L. ______________

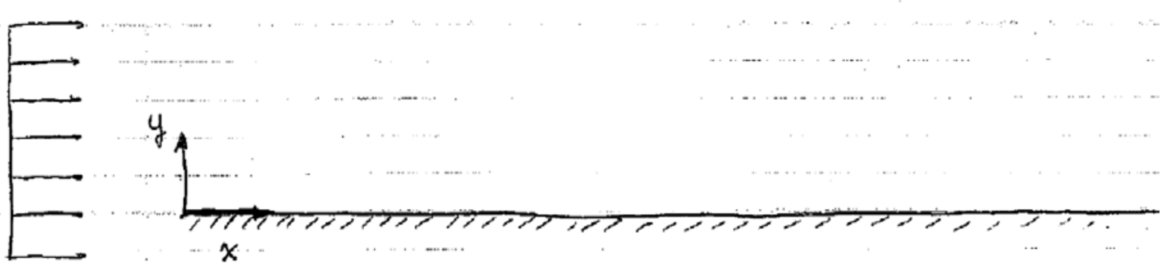
WHERE DOES TRANSITION TAKE PLACE?

$$\boxed{\begin{aligned} Re_x &= \underline{\qquad} = \underline{\qquad} \sim \\ &= \underline{\qquad\qquad} \end{aligned}}$$

⇑ FOR TRANSITION

REYNOLDS NUMBER

THERMAL BOUNDARY LAYER →



THE FUNDAMENTAL FLUIDS QUESTION . . .

THE FUNDAMENTAL HEAT TRANSFER QUESTION . . .

What is

(SO I CAN FIND

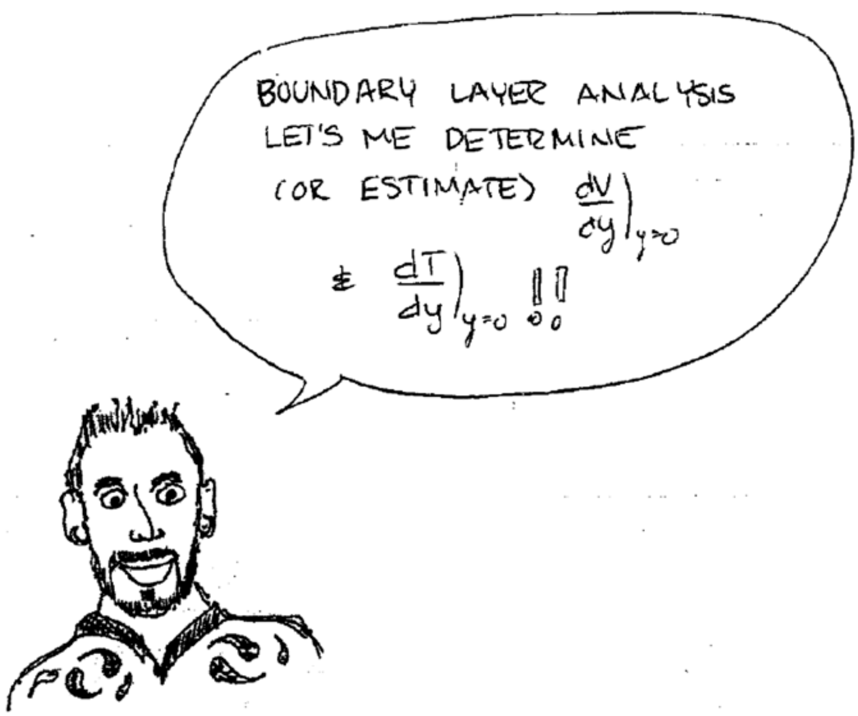
What is

(SO I CAN FIND

CAN BE ANSWERED IF . . .

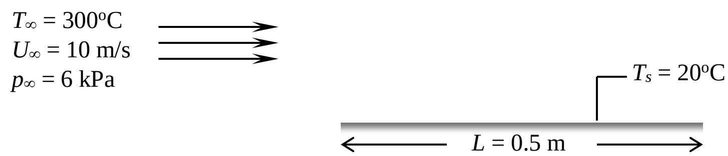
I KNOW

I KNOW



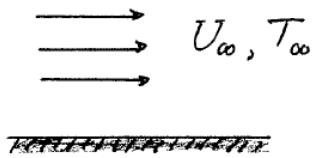
Example

Air at a pressure of 6 kPa and a temperature of 300°C flows with a velocity of 10 m/s over a plate of length 0.5 m. Estimate the cooling rate per unit width of the plate needed to maintain it at a surface temperature of 20°C.

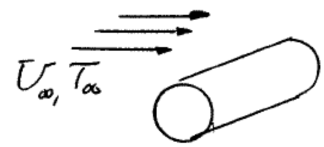


NOTES: External convection

BEFORE WE HAD

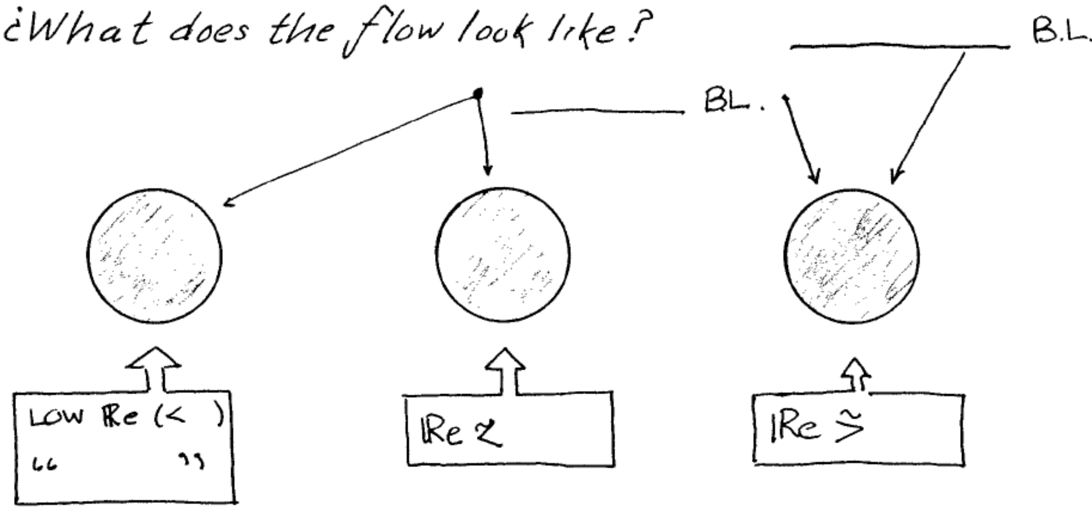


NOW WE HAVE



LET'S CONSIDER THE FLUID MECHANICS FIRST

¿What does the flow look like?



CAUTION

$$\mathrm{Re} = \frac{\rho U_\infty \square}{\mu} = \frac{U_\infty \square}{\nu}$$

FLAT PLATE:

$$F_D = C A \tfrac{1}{2} \rho U_\infty^2$$

WHICH AREA?

HERE

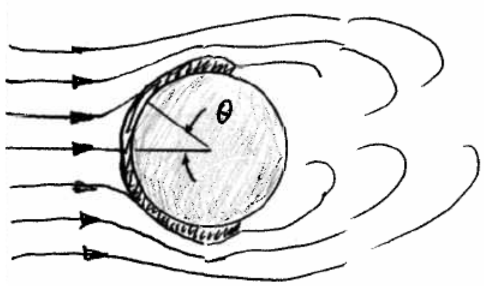
$$\boxed{F_D = C A \tfrac{1}{2} \rho U_\infty^2}$$

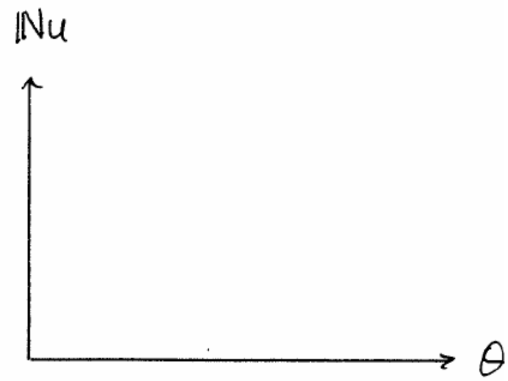
WHICH AREA?

FIG   GIVES C_D FOR CYLINDER & SPHERE (SMOOTH)

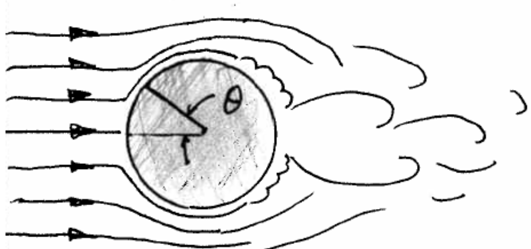
NOTES: External convection

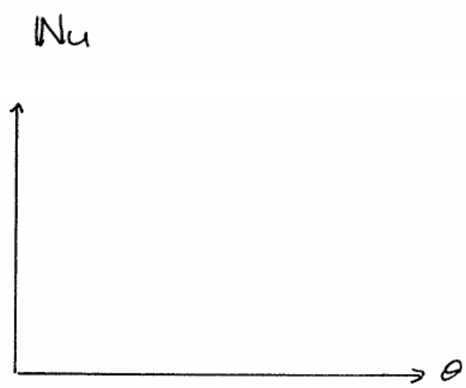
LOW Re





HIGH Re





$$Nu = \frac{h\,\square}{k}$$

$$Nu_{cyl} = 0.3 + \frac{\qquad\qquad}{[\qquad\qquad]}\left[1+\left(\frac{\quad}{\quad}\right)\right]$$

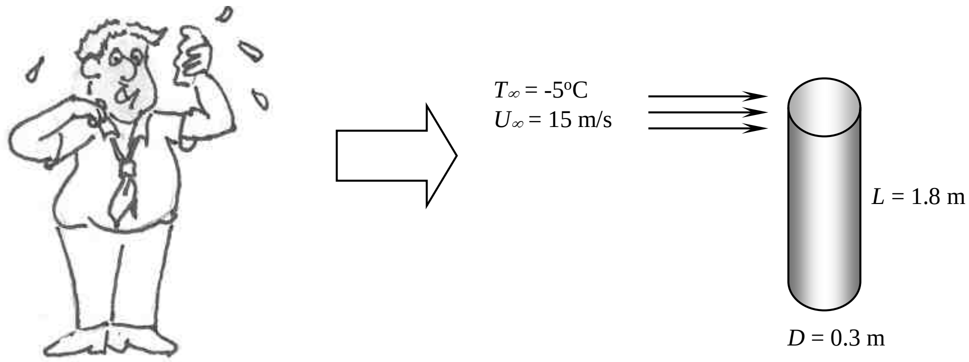
Like II

$$Nu_{cyl} =$$

Like I

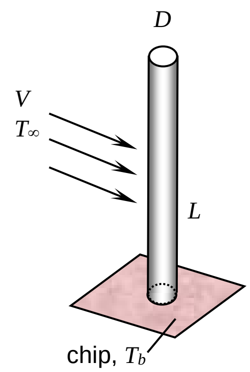
Example

Assume that a person can be approximated as a cylinder of 0.3-m diameter and 1.8 m height with a surface temperature of 25°C. Calculate the body heat loss while this person is subjected to a 15 m/s wind whose temperature is –5°C.



Example

To enhance heat transfer form a silicon chip, a copper pin fin is brazed to the surface of the chip. The pin length and diameter are L = 12 mm and D = 2 mm, respectively. The surface of the chip, and hence the base of the pin are maintained at a temperature of T_b = 350 K. The pin is subject to atmospheric air in cross flow with V = 10 m/s and T_∞ = 300 K



(a) What is the average convection coefficient for the surface of the pin?
(b) Assuming h at the tip of the fin to be the same as that calculated in a), calculate the heat transfer rate from the pin. (I.e., assume an insulated tip with a corrected fin length.)

EXERCISE: Find the correlation

1. A fluid flows past a flat plate of length L=1.0 m maintained at a constant temperature. The Reynolds number based on plate length is found to be Re=2.0×10^6 and the Prandtl number of the fluid is Pr=0.9. You wish to know the rate of heat transfer from the plate. What correlation for Nu do you use?

2. A fluid flows past a flat plate of length L=1.0 m maintained at a constant temperature. The Reynolds number based on plate length is found to be Re=2.0×10^4 and the Prandtl number of the fluid is Pr=0.9. You wish to know the rate of heat transfer from the plate. What correlation for Nu do you use?

3. A fluid flows past a flat plate of length L=1.0 m maintained at a constant temperature. The Reynolds number based on plate length is found to be Re=2.0×10^6 and the Prandtl number of the fluid is Pr=0.9. You wish to know the heat flux at the trailing edge of the plate, i.e., at x=L. What correlation for Nu do you use?

4. A fluid flows past a flat plate of length L=1.0 m maintained at a constant temperature. The Reynolds number based on plate length is found to be Re=2.0×10^5 and the Prandtl number of the fluid is Pr=0.9. You wish to know the rate of heat transfer from the plate. What correlation for Nu do you use?

5. A fluid flows past a flat plate of length L=1.0 m subject to a constant surface heat flux. The Reynolds number based on plate length is found to be Re=8.0×10^5 and the Prandtl number of the fluid is Pr=0.9. You wish to know the heat flux at the trailing edge of the plate, i.e., at x=L. What correlation for Nu do you use?

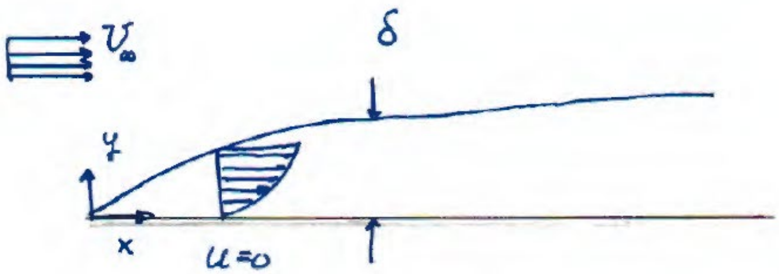
6. A fluid flows past a flat plate of length L=1.0 m maintained at a constant temperature. The Reynolds number based on plate length is found to be Re=8.0×10^5 and the Prandtl number of the fluid is Pr=0.9. You wish to know the heat flux at a location x=0.25 m from the leading edge of the plate. What correlation for Nu do you use?

7. A fluid flows past a flat plate of length L=1.0 m maintained at a constant temperature. The Reynolds number based on plate length is found to be Re=8.0×10^5 and the Prandtl number of the fluid is Pr=0.9. You wish to know total rate of heat transfer from the plate. What correlation for Nu do you use?

8. A fluid at temperature T_∞ flows past a flat plate of length L=1.0 m subject to a known constant surface heat flux $\dot{q}$. The Reynolds number based on plate length is found to be Re=2.0×10^5 and the Prandtl number of the fluid is Pr=0.9 You wish to know the surface temperature at the trailing edge of the plate, i.e., at x=L. What correlation for Nu do you use and how do you calculate the temperature?

NOTES: **The Prandtl number**

The Magic PRANDTL Number



<u>Fluids (momentum transfer)</u>

$$\tau_w = \mu \left.\frac{du}{dy}\right)_{y=0}$$

$$\sim \mu \, \frac{\quad}{\quad}$$

$$\frac{\delta}{L} \sim$$

<u>Heat transfer</u>

$$h = \frac{\quad}{\quad}$$

$$\sim$$

$$\mathbb{P}r \equiv \frac{\quad}{\quad} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$$

$$\frac{\delta_T}{\delta} \sim$$

NOTES: **The Prandtl number**

$$h \sim \frac{k}{\delta_T} \sim$$

$$\mathit{Nu} \sim$$

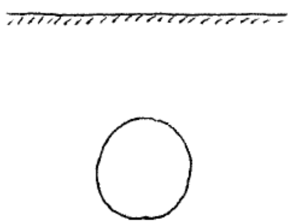
(At least for laminar flow on plates.
Other flows [turbulent, internal, etc.]
are more complicated.)

NOTES: Internal convection

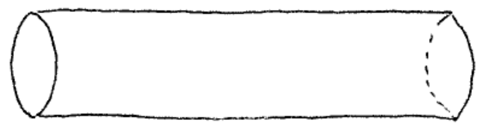
TWO TYPES OF Flow:

- EXTERNAL ____________________
- INTERNAL ____________________

EXTERNAL



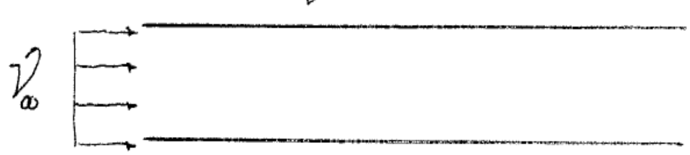
INTERNAL



INTERNAL FLOW

¿ How does internal flow differ from external flow in terms of boundary layers?

VELOCITY (MOMENTUM) BOUND. LAYER



Do you expect C_f (or f) & Nu to be higher in the developing region or the fully developed region? Why?

THERMAL BOUNDARY LAYER

YOU CAN SEE THAT $V = V(r)$ & $T_{\text{FLUID}} = T(r)_{\text{FLUID}}$ IN THE INTERNAL FLOW CASE. LET US DEFINE, THEN

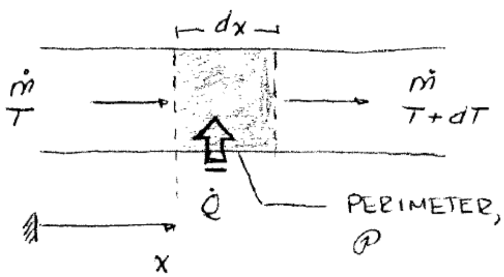
$$\boxed{\dot{m} = \rho V_m A}$$

$$\boxed{\dot{m} c_p T_m =}$$

OUR **GOAL** IS TO FIND $\dot{Q} = hA(T_s - T_m)$

¿ WHAT $T_s - T_m$ DO I USE ?

TAKE A SMALL SLICE of PIPE



Cons of Energy →

$$\frac{dE}{dt} = \dot{Q}_{IN} + \dot{W}_{in} + \sum_{in} \dot{m}(h + \ldots) - \sum_{OUT} \dot{m}(h + \ldots)$$

[EQN 1]

$\dot{q}_b$ ALSO $\Rightarrow$ $\dot{q}_b =$ [EQN 2]

COMBINING [1] & [2]

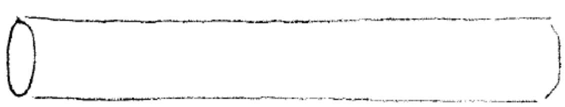
[EQN 3]

$\boxed{\text{CASE 1}}$ $\dot{q}_b$ = CONST

[1] SAYS

[2] SAYS
(IF h = CONST)

$\therefore$



$\dot{q}_b =$

$\dot{Q} =$

$\dot{q}_b$ = CONST
BOUNDARY CONDITION

CASE 2

T_s = CONST

[3] SAYS

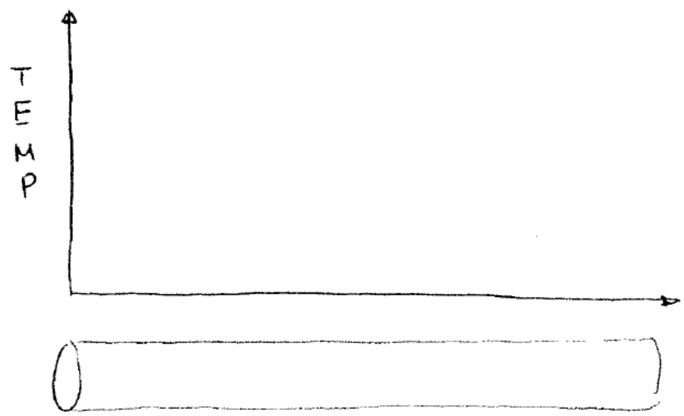
A LITTLE TRICK $\quad dT_m = -\,d(T_s - T_m)$

SO .

$\Rightarrow$ $T_m(x) =$

[EQN 5]

∴

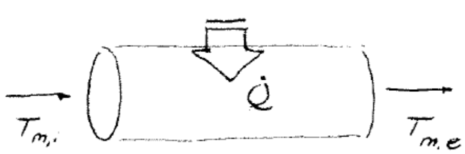


WE SEE THAT $\dot{Q} = hA(T_s - T_m)$ IS A PROBLEM.

LET'S USE $\dot{Q} = hA\,\Delta T_{AVG}$

¿ *What is* ΔT_{AVG}?

<u>*Cons. of Energy on whole tube* →</u>



$\dot{Q} =$ [EQN 6]

[5] FOR THE TUBE EXIT (@ x = L) GIVES

[EQN 7]

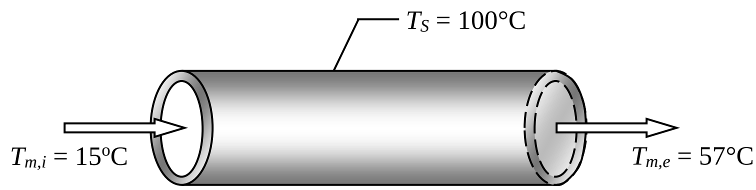
COMBINE [6] & [7] TO ELIMINATE $\dot{m}c_p$

$$\dot{Q} = hA \frac{\left[(T_s - T_{m,e}) - (T_s - T_{m,i})\right]}{\ln \dfrac{T_s - T_{m,e}}{T_s - T_{m,i}}}$$

$$\dot{Q} = hA$$

Example

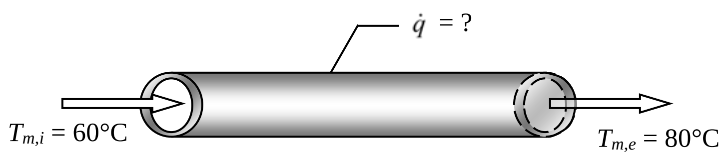
The average convection coefficient for water flowing through a circular tube is to be determined *experimentally*. In the experiment, steam condenses on the outer surface of a thin-walled circular tube with 50-mm diameter and 6-m length. This maintains the tube at a uniform surface temperature of 100°C. Water flows through inside the tube at a rate of $\dot{m}$ = 0.25 kg/s, and its inlet and outlet temperatures are $T_{m,i}$ = 15°C and $T_{m,e}$ = 57°C, respectively. What is the experimentally determined average convection coefficient associated with the water flow?



Example

Water flows through a section of 2.54-cm diameter tube 3.0 m long. The water enters the section at 60°C with a velocity of 2 cm/s. Assuming that the flow is **fully developed** (buzza buzza buzz) by the time it enters the region of interest and that the wall is subject to constant wall heat flux,

(a) calculate the wall heat flux (in W/m^2) needed to heat the water to 80°C.
(b) Calculate the wall temperatures at the inlet and the exit.
(c) Repeat part a) and b) if the velocity of the water is increased to 2 m/s.



Example

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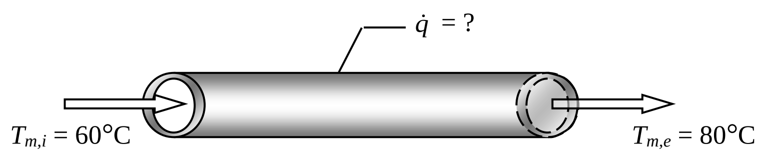
(a) ~~calculate the wall heat flux (in W/m²) needed to heat the water to 80°C.~~ **DONE!**
(b) ~~Calculate the wall temperatures at the inlet and the exit.~~ **DONE!**
(c) ~~Repeat part (a) and (b) if the velocity of the water is increased to 2 m/s.~~ **DONE!**
(d) Find the pressure drops and the pumping powers required for the two velocities above.

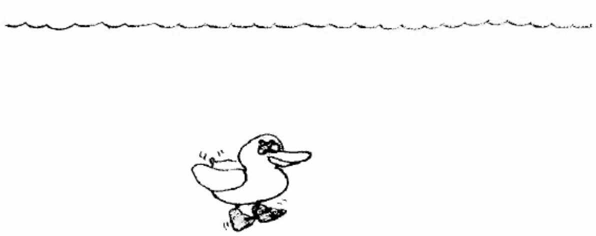
NEW!



NOTES: Natural convection

NATURAL CONVECTION & BUOYANCY

- IN FORCED CONVECTION FLUID MOTION IS CAUSED BY APPLIED PRESSURE GRADIENTS. THIS IS ACCOMPLISHED BY PUMPS, FANS, BLOWERS, ETC.
- IN NATURAL OR FREE CONVECTION, FLUID MOTION IS CAUSED BY ______________________.
- CONSIDER A RUBBER DUCKIE FLOATING BENEATH THE SURFACE OF A BATHTUB:



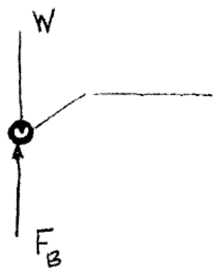
TWO FORCES ACT ON THE DUCKIE:

THE *NET UPWARD FORCE* IS, THEN

$F_{NET} =$ $\qquad\qquad\qquad =$

NOTES: Natural convection

NOW RATHER THAN A RUBBER DUCKIE, LET'S SAY YOU'VE GOT A FLUID PARTICLE THAT'S ____________ IT A MEDIUM THAT'S ________



$F_{NET,UP} =$

WHAT DO YOU KNOW ABOUT ρ OF HOT FLUIDS COMPARED TO ρ OF COLD FLUIDS?

WE SEE, THEN THAT

____________________ CAUSE

____________________ CAUSE

____________________ .

AND WHERE THERE'S ____________________ THERE'S CONVECTION.

THAT'S NATURAL CONVECTION!

A CLOSER LOOK AT $\Delta\rho$:

$\boxed{\beta \equiv \qquad}$

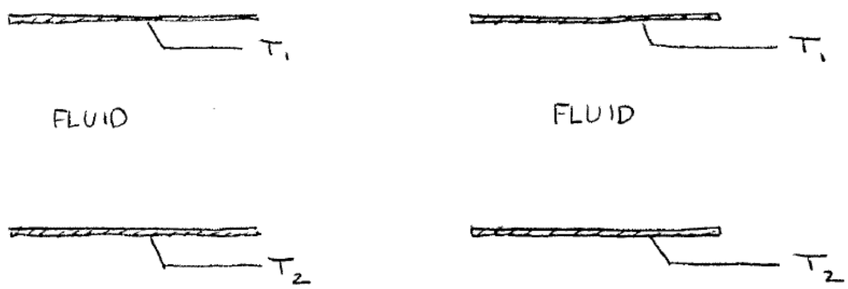
$\beta \approx$ $\qquad\qquad$ $\therefore \Delta\rho \approx$

NOTES: Natural convection

$F_{NET,UP} \approx$

- IF $T > T_\infty$
- IF $T < T_\infty$

CONSIDER TWO PLATES SEPARATED BY AN INITIALLY STILL FLUID.



(a) (b)

SUDDENLY WE HEAT ONE of THE PLATES. IN (a) WE HEAT THE TOP PLATE SUCH THAT $T_1 > T_2$. IN (b), $T_2 > T_1$.

What happens?

IF BUOYANCY MOVES FLUID, WHAT OPPOSES THE MOTION?*

__________________.

LET'S DEFINE A DIMENSIONLESS NUMBER THAT MEASURES THE RELATIVE IMPORTANCE of THESE FORCES:

$Gr = \_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_\_\_$

$= \_\_\_\_\_\_\_\_\_\_$

$\boxed{Gr = \qquad\qquad}$

HIGH Gr MEANS __________________.

LOW Gr MEANS __________________.

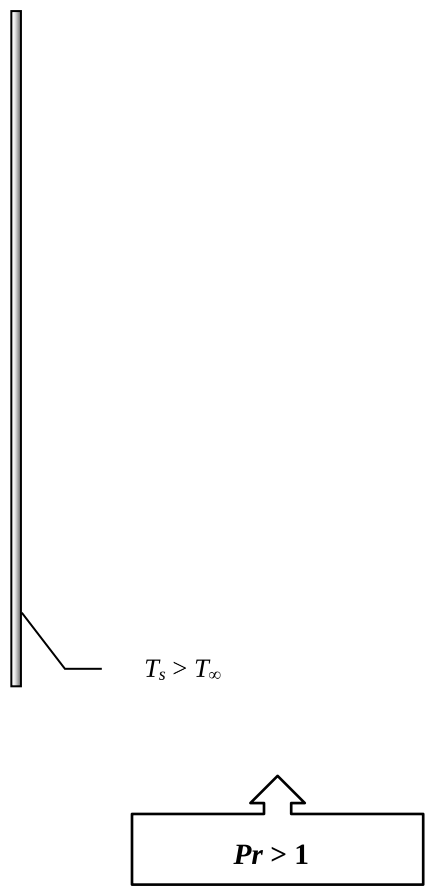
* (STRICTLY TRUE FOR Pr > 1.)

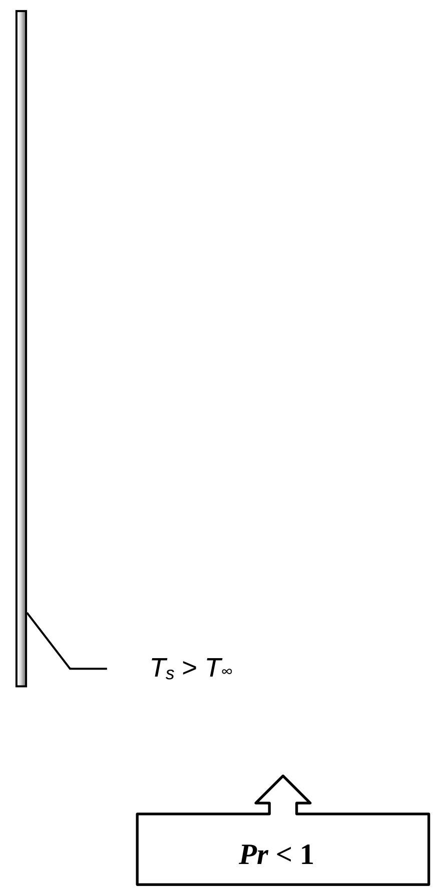
ACTIVE LEARNING EXERCISE—Natural convection boundary layers

Remember that one interpretation of Prandtl number is a measure of the relative thickness of a momentum (velocity) boundary layer to a thermal boundary layer. With this thought in mind,

1. sketch the momentum and thermal boundary layers for natural convection on a vertical wall with $T_s > T_\infty$ if $Pr > 1$. Include the variation of velocity and temperature across the layers.

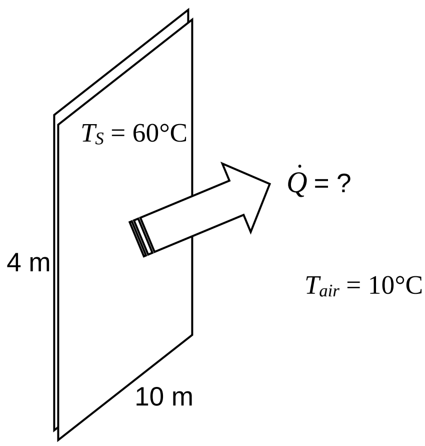
2. Sketch the momentum and thermal boundary layers for natural convection on a vertical wall with $T_s > T_\infty$ if $Pr < 1$. Include the variation of velocity and temperature across the layers.





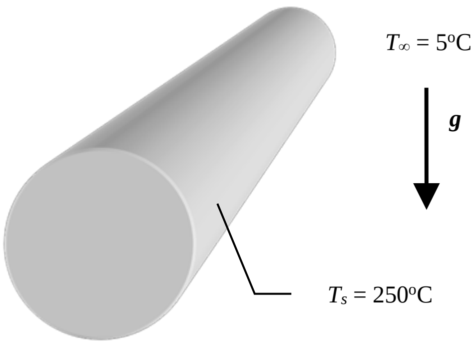
Example

A large vertical plate 4.0 m high is maintained at 60°C and exposed to atmospheric air at 10°C. Calculate the heat transfer rate from the plate if it is 10 m wide.



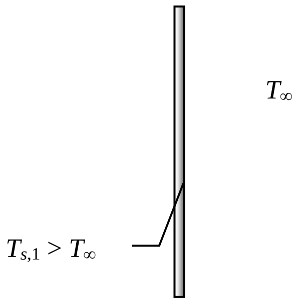
Example

The surface of a horizontal pipe 1 ft (0.3048 m) in diameter is maintained at a temperature of 250°C in a room where the ambient air is at 15°C. Calculate the free-convection heat loss per meter of length.

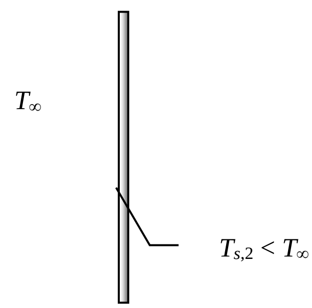


ACTIVE LEARNING EXERCISE—Natural convection in enclosures

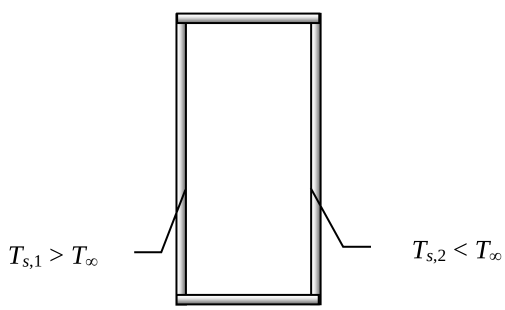
1. Imagine a vertical plate at a temperature $T_{s,1}$ in a quiescent fluid at T_∞. Assuming that $T_{s,1} > T_\infty$, sketch the velocity boundary layer that forms as a result of the temperature-induced density gradients next to the wall.



2. Now imagine a vertical plate at a temperature $T_{s,1}$ in a quiescent fluid at T_∞, but this time assume that $T_{s,1} < T_\infty$, sketch the velocity boundary layer that forms as a result of the temperature-induced density gradients next to the wall.

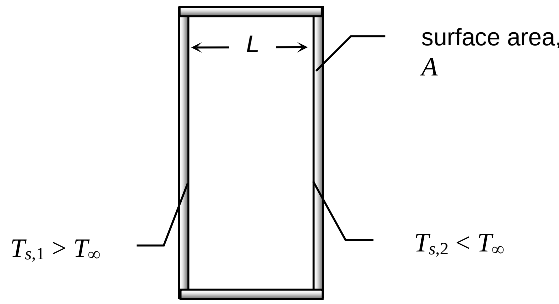


3. Let us bring the two vertical plates close to each other, and then cap the top and bottom to form an ____________________. Sketch what you think the flow pattern of fluid would look like in the enclosure.



4. We know the fluid is not stationary, but if it were, what would be the mode of heat transfer between the walls?

5. For steady state, write an expression for the rate of heat transfer between the two walls *assuming no fluid motion*.



6. Since there really is fluid motion, we know the mode of heat transfer is ______________. Does it make since to use ($T_{s,1}$ - T_∞) as the temperature difference for the total heat transfer rate across the entire enclosure? What about ($T_{s,2}$ - T_∞)? What temperature difference *does* make sense to use? What would your expression for the rate of heat transfer look like, then?

7. We can still calculate the rate of heat transfer assuming we have steady-state, 1-D conduction as in part 5., *if* we use a pretend, **effective conductivity** of the fluid.

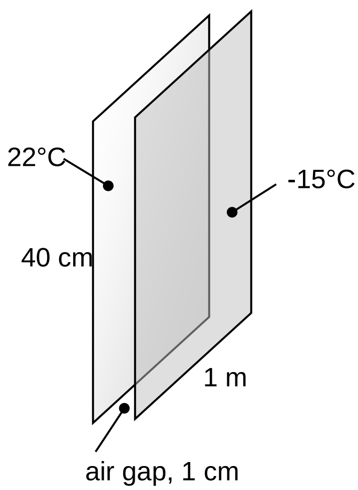
   This pretend conductivity is larger/smaller than the actual conductivity due to the fluid motion. (circle one)

   And so finally, equate your expressions for heat transfer rate in parts 5. and 6., but write the equation and solve it for the effective thermal conductivity of the fluid. (Hint, remember that $Nu = hL_{chr}/k$ where k is the real thermal conductivity of the fluid.

Example

A double pane window is 40 cm high and 1 m wide. The air gap between the two pieces of glass is 1 cm. The inside and outside temperatures of the window are 22°C and -15°C, respectively. Neglecting the thermal resistance of the glass,

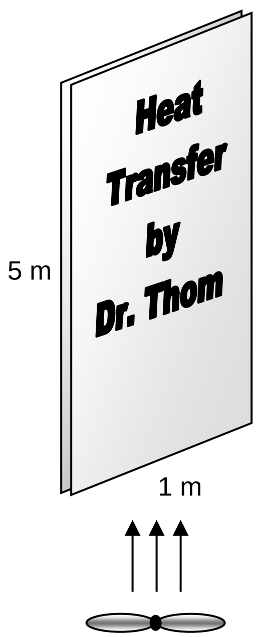
(a) calculate the rate of heat transfer through the glass ignoring the effects of natural convection; i.e., if heat transfer is by conduction only.
(b) Calculate the rate of heat transfer through the window considering natural convection.
(c) Repeat part b) if the gap thickness is increased to 2 cm. Discuss the results.



Example

In a fit of temporary insanity, a frustrated Rose student painted a piece of ply wood to resemble a giant novelty-sized heat transfer book, took it to the front lawn, and set it on fire. Luckily, the fire was put out quickly and no one was hurt. Sometime after the fire was put out, it was observed that the "book" temperature was 85°C and the surrounding air temperature was 29°C. A small fan was placed beneath the "book" to aid in its cooling.

(a) Determine the minimum air velocity for which natural convection is negligible.
(b) Find the rate of heat transfer from the "book" if the air velocity is 5 m/s.



ACTIVE LEARNING EXERCISE – Non-dimensionalization

Remember the velocity (momentum) boundary layer equation (conservation of ________________ applied at ____ _________ _________ within the boundary layer)?

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}$$

Now if we have *buoyancy* as well, we have to add a buoyancy term:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}= \qquad\qquad\qquad +\nu\frac{\partial^2 u}{\partial y^2}$$

Non-dimensionalization gives us a way to weigh the relative importance of different physical phenomena. One way to arrive at these dimensionless groups is to use the **Buckingham Pi Theorem** to derive the dimensionless groups, or pi terms, directly. Another way is to define dimensionless versions of the variables which show up in the working equations, and then to substitute those variables into the equations. For example, a dimensionless version of the x-direction velocity, u is given by:

$$u^* = u/U_\infty$$

Wherever the variable u shows up in the boundary layer equation, then, we would substitute u^*U_∞ instead.

Let us continue with this idea by defining dimensionless versions of the rest of the variables and substituting…

Radiation terms

Radiation heat transfer lingo is bountiful. To make matters worse, many of these terms seem like they should mean the same thing, but actually refer to different concepts. Below is a list of some of these terms. You are encouraged to write the definitions of these terms as you come across them in the readings. *A clear understanding of what these terms mean will make your study of radiation go more smoothly.*

ABSORPTIVITY

BLACK BODY

DIFFUSE

DIRECTIONAL

EMISSIVE POWER

EMMISIVITY

GRAY

IRRADIATION

(MORE ON BACK)

OPAQUE

RADIATION

RADITIATION INTENSITY

RADIOSITY

REFLECTIVITY

RERADIATING SURFACE

SHAPE (VIEW) FACTOR

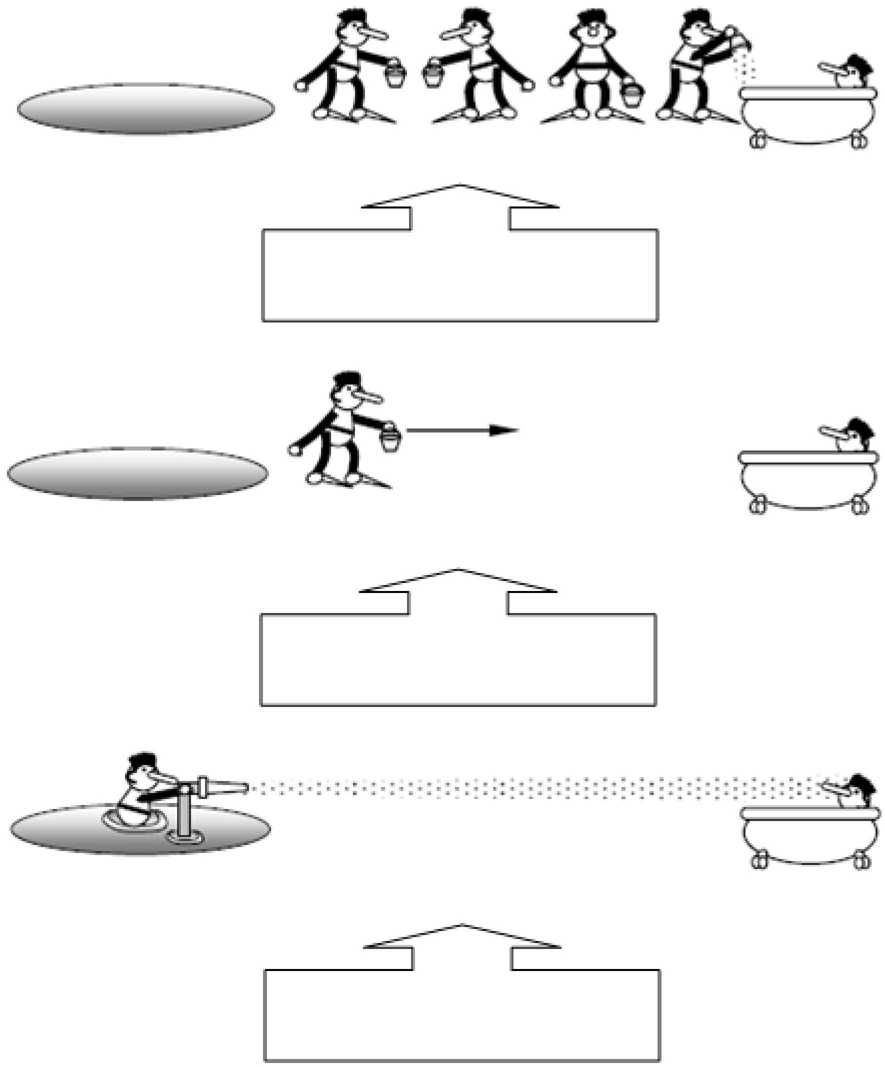
SPECTRAL

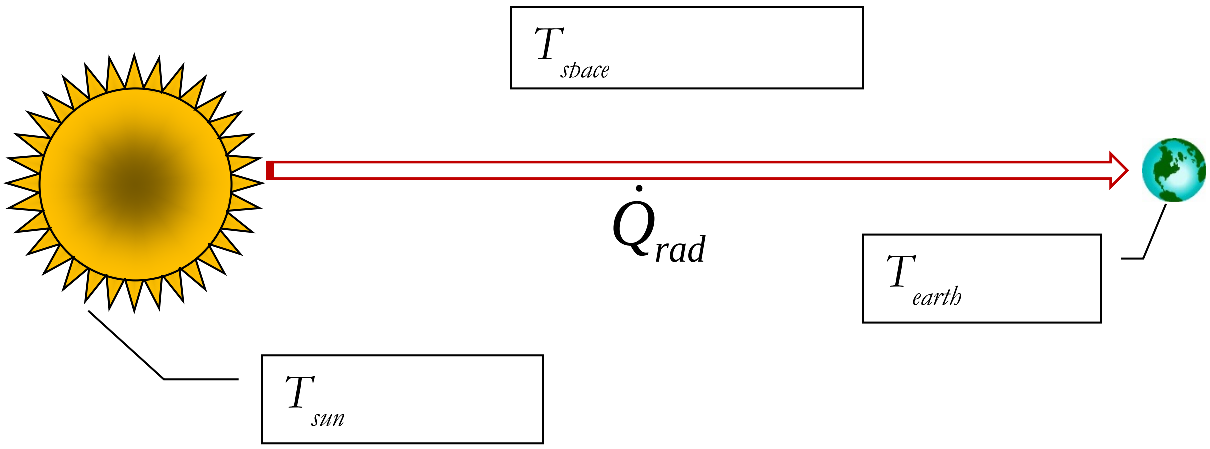
TOTAL, TOTAL HEMISPHERICAL

TRANSMISIVITY

NOTES: Intro to radiation

Introduction to Radiation



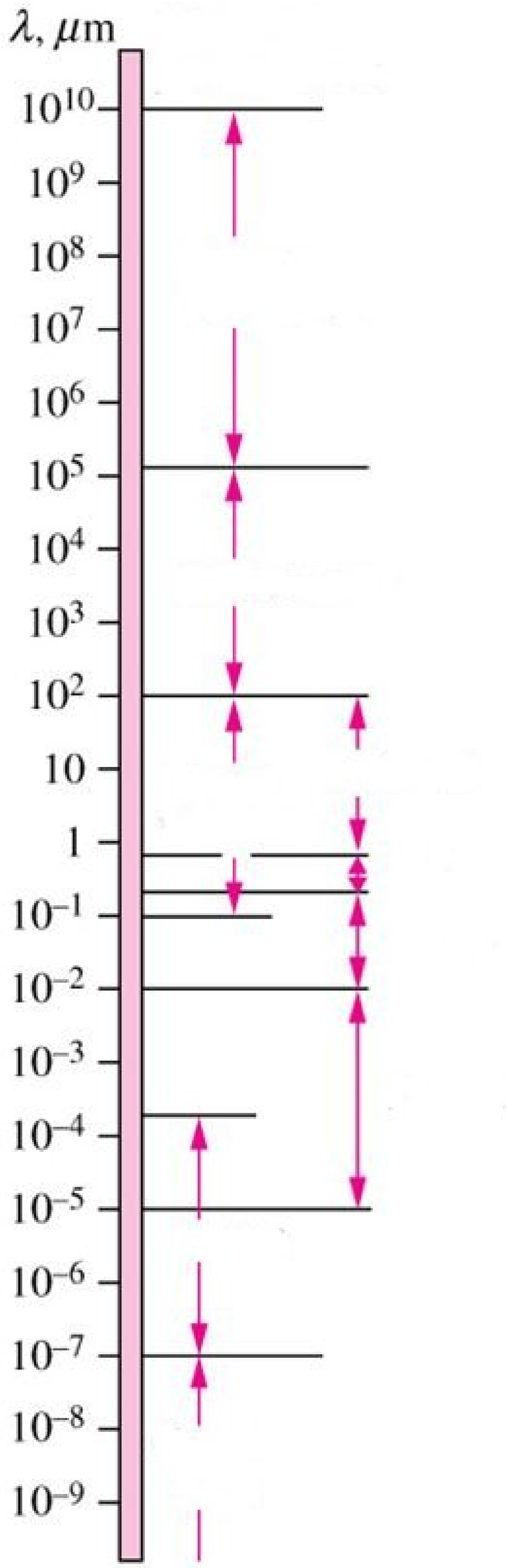


Radiation does _________________, but it can go through one, *even if*

Radiation is a _________ **phenomenon**.

I.e., radiation heat transfer occurs as an exchange ____________________.

Types of radiation as a function of wavelength



NOTES: Blackbody radiation

BLACKBODY RADIATION

A blackbody is an idealized ____________

- No surface ________ more.*
- No surface ________ more.
- Emits same amount of radiation in ____ ________ ⇨ It is ________.

Black surface   Real surface

EMISSIVE POWER

Heat transfer ______________ by a surface ____ ____ ______ of that surface.

$E_b =$ (Blackbody)

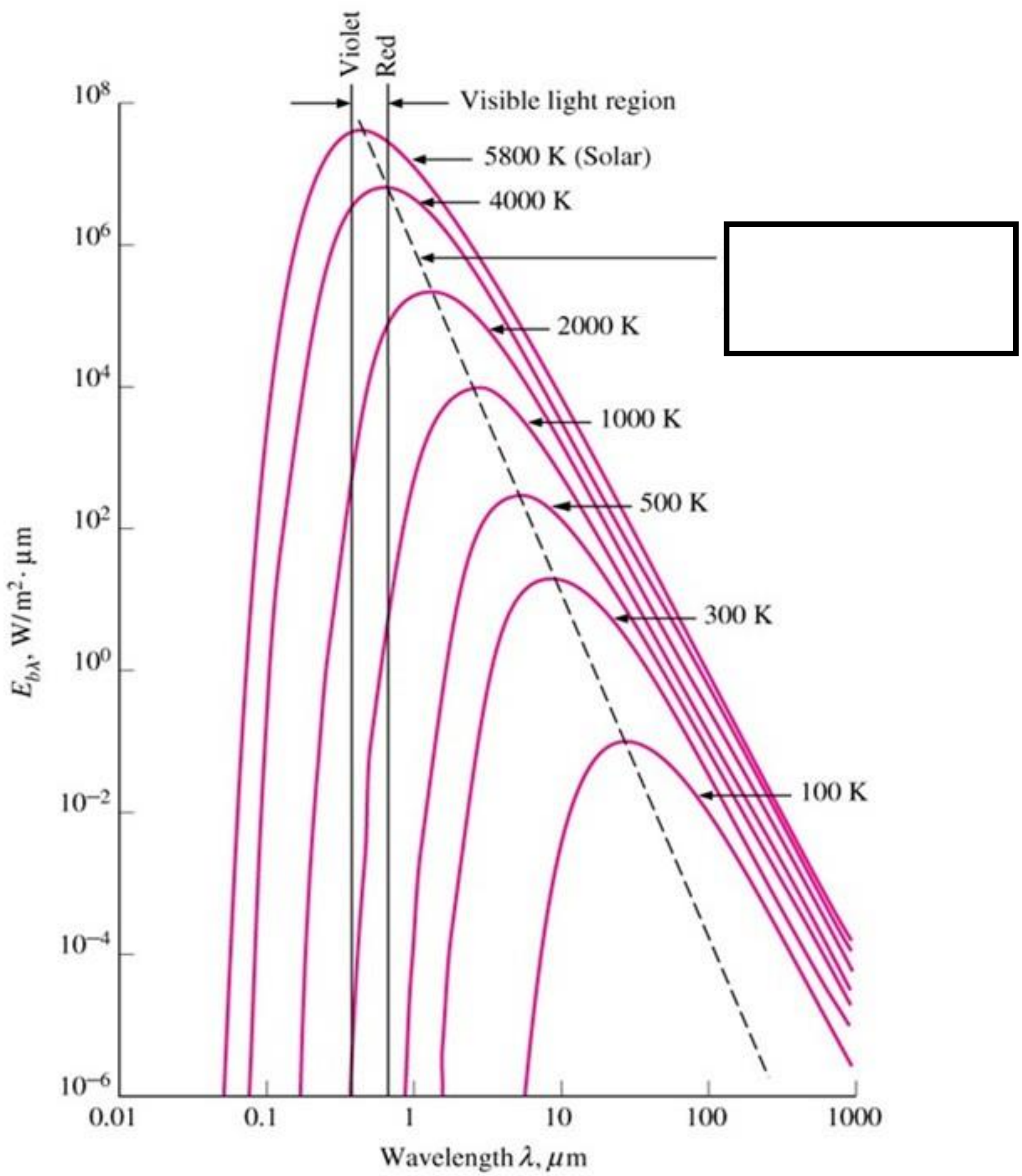
$E =$ (Real surface)

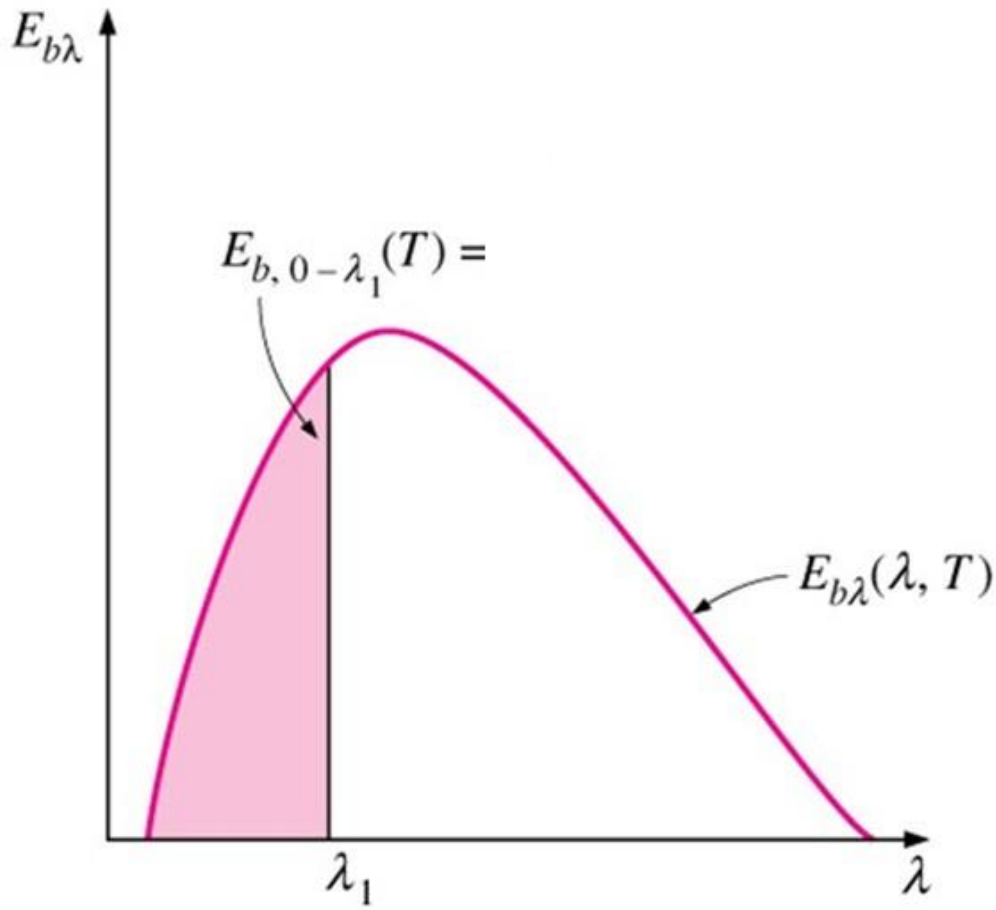
---

* what is ε for a blackbody?

NOTES: Blackbody radiation

Blackbody Emissive Power



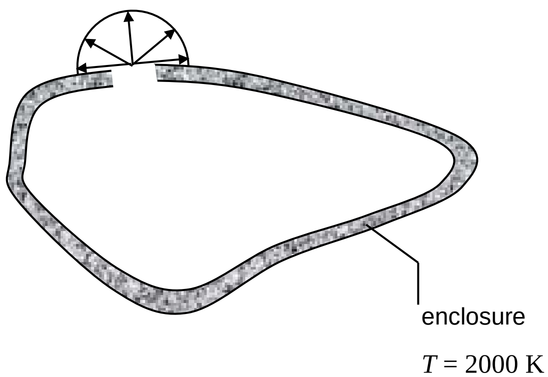


$E_{b\lambda}$
$E_{b,0-\lambda_1}(T) =$
$E_{b\lambda}(\lambda, T)$
λ_1
λ

Example

Consider a large, isothermal enclosure that is maintained at a uniform temperature of 2000 K.

(a) Calculate the emissive power of the radiation that emerges from a small aperture on the surface.
(b) What is the wavelength below which 10% of the emission is concentrated?
(c) What is the wavelength above which 10% of the radiation is concentrated?
(d) Determine the maximum spectral emissive power and the wavelength at which it occurs.



RADIATION PROPERTIES

FOR BLACKBODIES

$$E_b(T) = \sigma T^4$$

____, ____ EMISSIVE P

ALL ____

ALL ____

$$\doteq \frac{[\quad]}{[\quad]}$$

A REAL BODY EMITS LESS

$$E(T) = \epsilon E_b = \epsilon \sigma T^4$$

-OR-

$$\boxed{\epsilon \equiv \frac{E(T)}{E_b(T)}}$$

(,)

@ A PARTICULAR WAVELENGTH, PER UNIT WAVELENGTH

$$\boxed{\epsilon_\lambda \equiv \frac{E_\lambda(T)}{E_{b,\lambda}(T)}}$$

IF A SURFACE IS

____, ITS PROPERTIES ARE INDEPENDENT of

____, ITS PROPERTIES ARE INDEPENDENT of

CAN ALSO DEFINE

ϵ_θ:

$\epsilon_{\lambda,\theta}$:

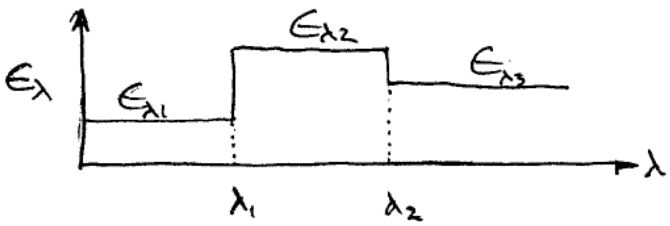
FOR A <u>NON</u>-GRAY SURFACE

$$E = E_{b,\lambda} = \sigma T^4$$

SO

$$\epsilon(T) = \frac{\int_0^\infty \epsilon_\lambda E_{b,\lambda}\, d\lambda}{\sigma T^4}$$

FOR ϵ_λ THAT VARIES IN A STEP-LIKE FASHION; i.e.



$$\epsilon = \frac{\int_0^{\lambda_1} \epsilon_{\lambda_1} E_{b,\lambda}\, d\lambda}{\sigma T^4} + \epsilon_{\lambda_2} \frac{\int_{\lambda_1}^{\lambda_2} E_{b,\lambda}\, d\lambda}{\sigma T^4} + \dots$$

THUS

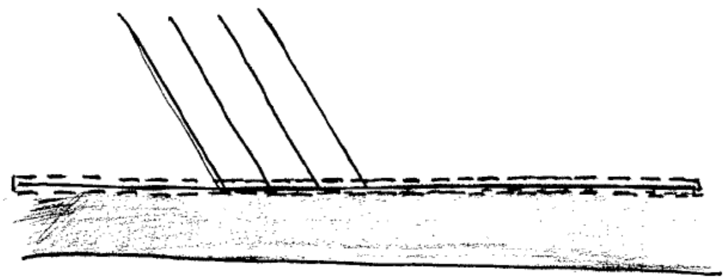
$$\boxed{\epsilon = \epsilon_{\lambda,1} \qquad + \qquad \epsilon_{\lambda,2} \qquad + \dots}$$

¡LIST THE ASSUMPTIONS!

NOTES: Radiation properties

OTHER PROPERTIES

INCIDENT RADIATION (   )



$\alpha = \dfrac{\_\_\_\_\_\_\_\_}{\text{INCIDENT}} = \_\_\_$ *

$\rho = \dfrac{\_\_\_\_\_\_\_\_}{\text{INCIDENT}} = \_\_\_$ *

$\tau = \dfrac{\_\_\_\_\_\_\_\_}{\text{INCIDENT}} = \_\_\_$ *

CONS. of ENERGY ON THIS SURFACE REQUIRES

$$G + G + G = G$$

-OR-

(,   )

IF A SURFACE IS ________

$\tau = 0 \Rightarrow$

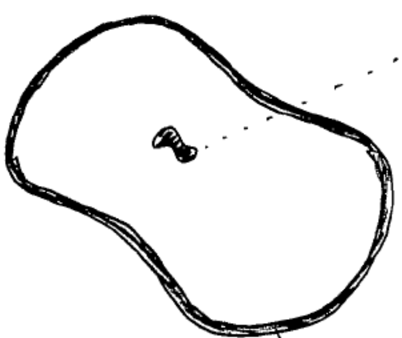
* NOTE THAT THESE PROPS. NOT ONLY DEPEND ON THE SURFACE, BUT ALSO THE SOURCE of THE IRRADIATION!!

IF SOURCE of G IS A BLACKBODY & α VARIES STEPWISE

$$\alpha = \alpha_{\lambda_1} \underbrace{\frac{\int_0^{\lambda_1} E_\lambda \, d\lambda}{\sigma T_{SOURCE}^4}} + \alpha_{\lambda_2} \frac{\int_{\lambda_1}^{\lambda_2} E_\lambda \, d\lambda}{\sigma T_{SOURCE}^4} + \ldots$$

$$= \alpha_{\lambda_1} \qquad\qquad + $$

KIRCHOF'S LAW



CONS. of ENERGY

$$\frac{dE_{sys}}{dt} =$$

LARGE, ISOTHERMAL ENCLOSURE @

($\therefore$)

*

WARNING! IF $T_{SOURCE} \gg T$

OR

$T_{SOURCE} \ll T$

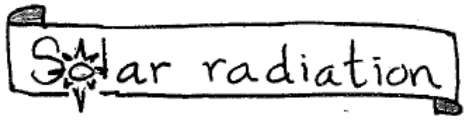
$\neq$!!

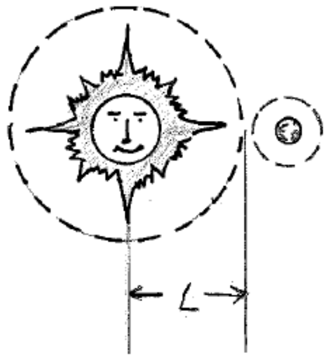
* FOR RESTRICTIONS ON $\alpha_{\lambda,\theta} = \epsilon_{\lambda,\theta}$

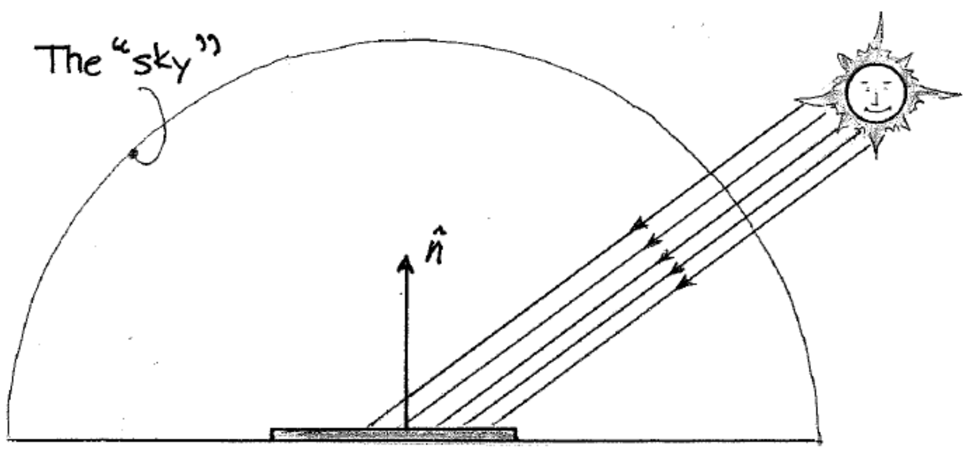
Example

The reflectivity of aluminum coated with lead sulfate is 0.35 for radiation at wavelengths less than 3 μm and 0.95 for radiation greater than 3 μm. (This is the **spectral** reflectivity.)

(a) Determine the average absorptivity of this surface for solar radiation. (T = 5800 K). Assume that the **incident radiation is well approximated by black body radiation**. (Hint: Can you relate reflectivity to the absorptivity?)
(b) Determine the absorptivity of the surface for radiation coming from sources at room temperature (T = 300 K). Ditto on the B-B stuff, and the hint too.
(c) Determine the emissivity of the surface at 300 K. Based on your results, would this be good stuff to use for solar collectors? Why or why not?


Solar radiation

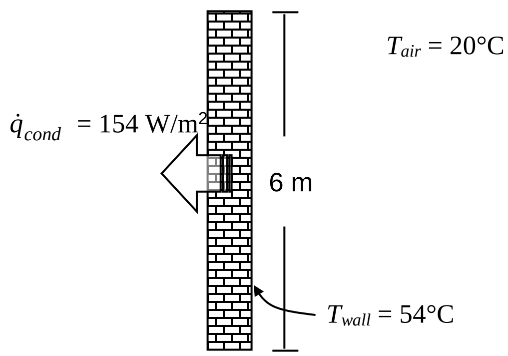

L


The "sky"
$\hat{n}$

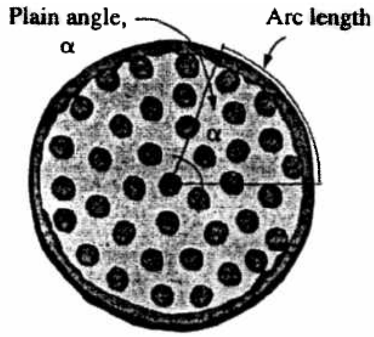
Example

The wall of a 6-m tall building is made of red brick, for which the emissivity, ε, is 0.93 and the *solar* absorptivity, α_s, is 0.63. On a sunny day, it is observed that the direct and diffuse components of solar radiation are G_D = 900 W/m² and G_d = 500 W/m², respectively, and that the sun makes a 48.2° angle with a normal to the surface of the wall. The outside temperature of the brick is 54°C, and the ambient air temperature is 20°C.

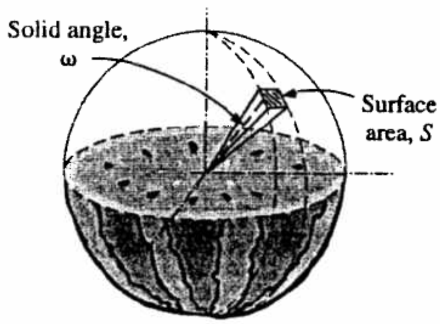
(a) Calculate the heat flux, in W/m², from the wall due to convection.
(b) If the heat flux through the brick due to conduction is 154 W/m² (into the building), what is the effective sky temperature?



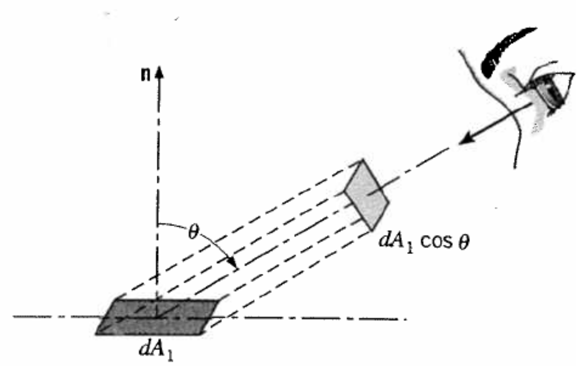
NOTES: View factors



A slice of pizza of plain angle α



A slice of watermelon of solid angle ω



SOLID ANGLE

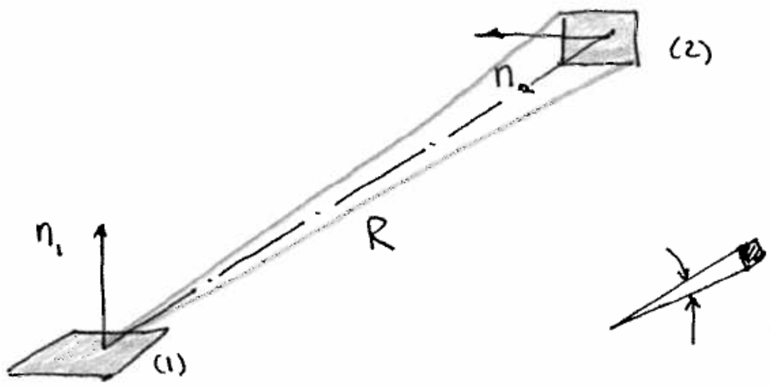
$d\omega =$

RADIATION INTENSITY

$I_e(\theta, \phi) =$

$dE = \dfrac{\quad}{\quad}$

DIFFUSELY EMITTING SURFACE



$d\omega_{1-2} =$ ______

$F_{1\to 2} =$ FRACTION of RAD. LEAVING (1) INCIDENT ON (2)

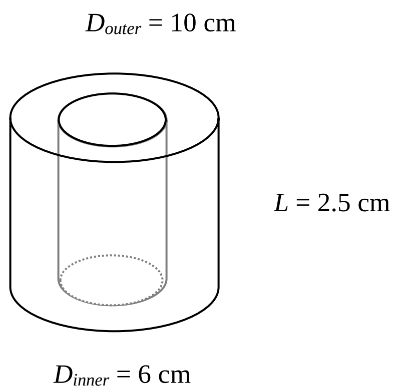
$$d\dot{Q}_{1\to 2} =$$

$$\boxed{F_{1\to 2} = \frac{1}{A}\iint \underline{\hspace{6em}}}$$

Example

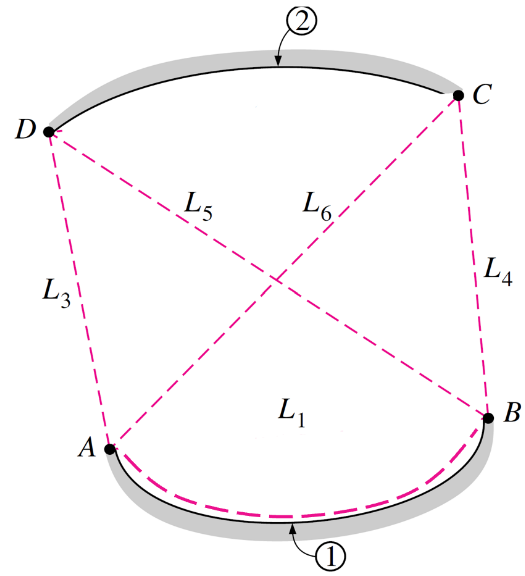
Two concentric cylinders are nested together coaxially as shown in the figure. Assuming the surfaces are *diffuse*,

(a) calculate the fraction of radiation leaving the outer surface of the inner cylinder that goes through the top and bottom openings.
(b) Calculate the fraction of radiation leaving the outer surface of the inner cylinder that goes through just the top opening.
(c) Calculate the fraction of radiation leaving the inner surface of the outer cylinder that goes through the top and bottom openings.



NOTES: **Crossed string method**

The crossed string method



Assumptions:

1.

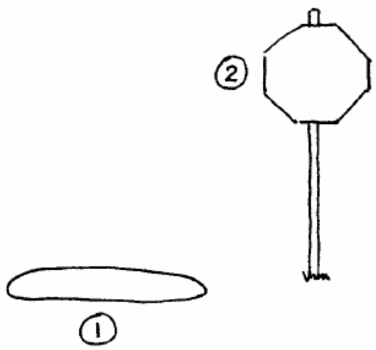
2.

3.

NOTES: Radiation between black surfaces

RADIATION EXCHANGE BETWEEN BLACK SURFACES

WRITE AN EXPRESSION FOR THE <u>NET</u> RATE of RADIATION HEAT TRANSFER FROM ① TO ②. ASSUME BOTH SURFACES ARE BLACK.



$$\dot{Q}_{NET,1\to2} = \underline{\qquad\qquad} - \underline{\qquad\qquad}$$

REARRANGE $\dot{Q}_{NET,1\to2}$ IN THIS FORM:

$$\boxed{\dot{Q}_{1\to2} = \frac{E_{b,1} - E_{b,2}}{\boxed{\qquad}}}$$

$\boxed{\qquad\qquad\qquad\qquad}$

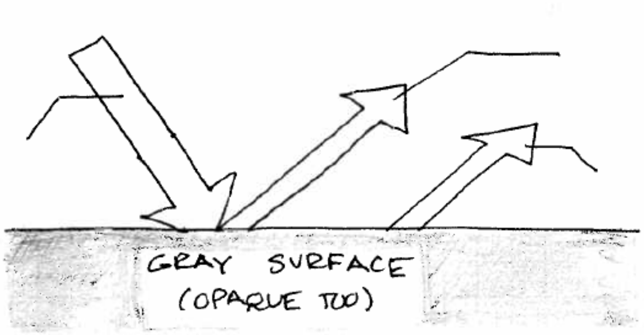
IF SURFACES FOR AN ENCLOSURE

$$\dot{Q}_{i,NET} =$$

$\dot{Q}$

NOTES: Radiation between black surfaces

RADIATION EXCHANGE BETWEEN GRAY SURFACES



$\equiv$ ____________

$J =$ ____________

PUT IN TERMS of ϵ, E_b & G

$J =$ *

WRITE AN EXPRESSION FOR THE NET RATE of RADIATION HEAT TRANSFER LEAVING A GRAY SURFACE.

$\dot{Q}_{i,NET} =$ ________ $-$ ________ $=$

ELIMINATE G

$\dot{Q}_{i,NET} =$

REARRANGE IN THIS FORM:

$$\dot{Q}_{i,NET} = \frac{E_{b_i} - J_i}{\boxed{}}$$

* WHAT IS J FOR A BLACKBODY?

NOTES: Radiation between black surfaces

BLACK SURFACE

$\dot{Q}_{NET} =$

$\dot{Q}_{NET} =$

GRAY SURFACE

$\dot{Q}_{NET} =$

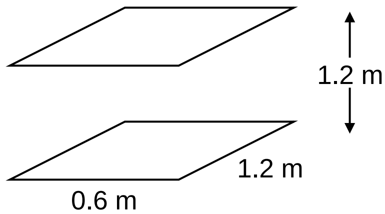
$\dot{Q}_{NET}$

Example

Two blackbody rectangles, 0.6 m by 1.2 m, are parallel and directly opposed. The bottom rectangle is at T_1 = 500 K and the top rectangle is at T_2 = 900 K. The two rectangles are 1.2 m apart.

(a) Find the view factors $F_{1\text{->}2}$ and $F_{2\text{->}1}$.
(b) Find the radiant exchange *between* the two surfaces.
(c) Find the rate at which the bottom rectangle is losing energy if the surroundings (other than the top rectangle) are considered to be a blackbody at 300 K.

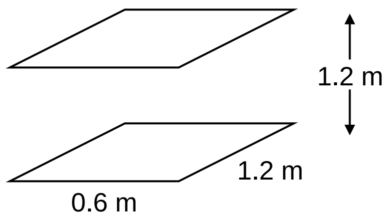
For the heat transfer calculations, you are strongly encouraged to draw all relevant resistors and currents (heat transfer rates).



Example

Reconsider the last example, but this time assume the surfaces are both diffuse and gray with $\varepsilon_1 = \varepsilon_2 = 0.7$. Otherwise, the conditions are the same. (The bottom rectangle is at T_1 = 500 K and the top rectangle is at T_2 = 900 K. The two rectangles are 1.2 m apart. The surroundings can be considered a blackbody at 300 K.)

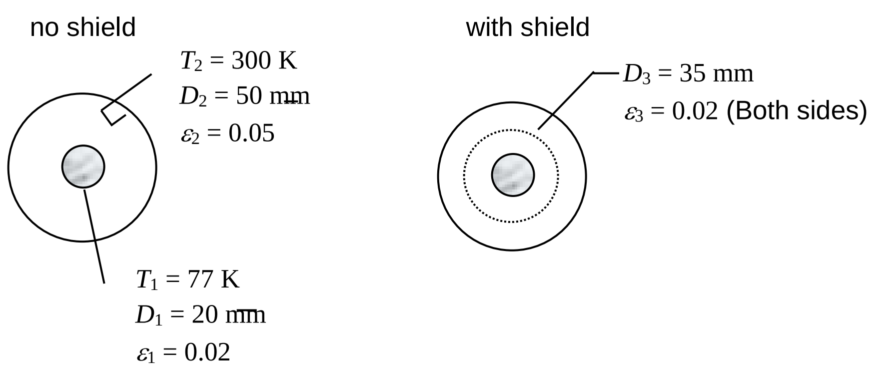
(a) Draw a resistance network showing all the relevant heat transfer rates and resistances.
(b) Find the net radiant exchange *between* the two surfaces.
(c) Find the rate at which the bottom rectangle is losing energy.
(d) Repeat (b) and (c) if the surroundings are treated as a **reradiating surface** instead.



Example

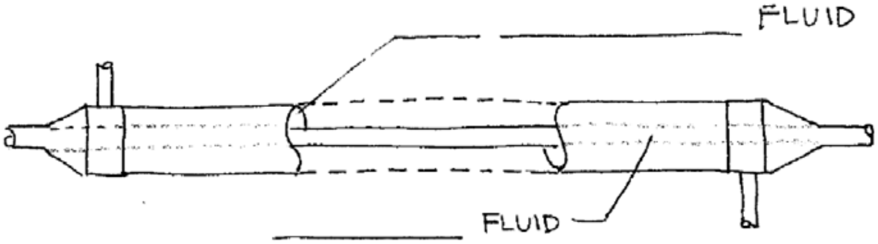
A cryogenic fluid flows through a long tube of 20 mm diameter, the outer surface of which is diffuse and gray with ε_1 = 0.02 and T_1 = 77 K. (Ooh, that's cold!) The tube is concentric with a larger tube of 50 mm diameter, the inner surface of which is diffuse and gray with ε_2 = 0.05 and T_2 = 300 K. The space between the surfaces is evacuated. If the tube is 1 m long (into the paper)

(a) calculate the heat gain by the cryogenic fluid.
(b) If a thin radiation shield of 35 mm diameter and ε_3 = 0.02 on both sides is inserted mid-way between the inner and outer surfaces, calculate the heat gain by the cryogenic fluid. What is the percentage change in heat gain?



NOTES: Heat exchangers

DOUBLE PIPE (DOUBLE PIPE) HEAT EXCHANGER



NOTATION: T_h :

T_c :

(IN) :

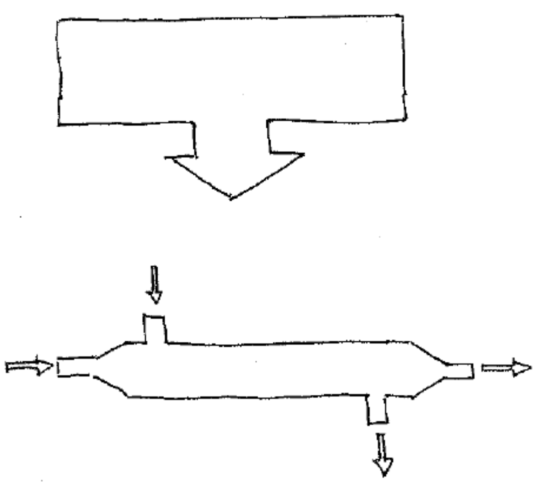
(OUT) :

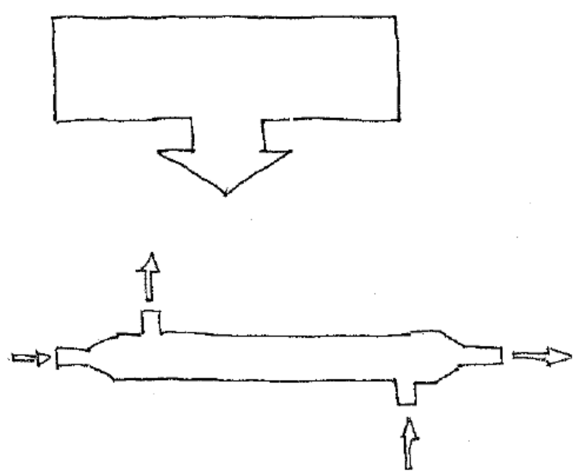
E.G.,

$T_{c,out} =$

$T_{h,in} =$

TWO ARRANGEMENTS





ASSUMPTIONS FOR ANALYSIS:

1)

2)

3)

4)

WE WOULD LIKE A HEAT TRANSFER COEFFICIENT THAT GIVES $\dot{Q}$ BETWEEN THE TWO FLUIDS FOR THE WHOLE HXR*.

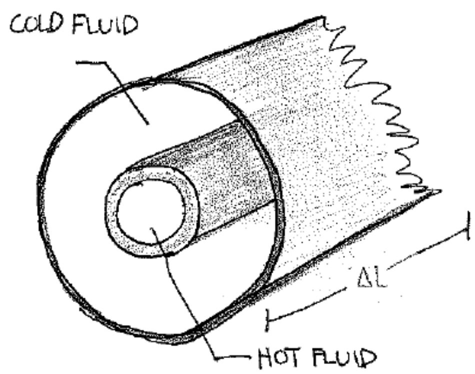
¿ $\dot{Q} =$ _____ _____ _____ ?

HEAT TRANSFER COEFFICIENT ↑ | AREA ↑ | TEMPERATURE DIFFERENCE ↑

_____ _____

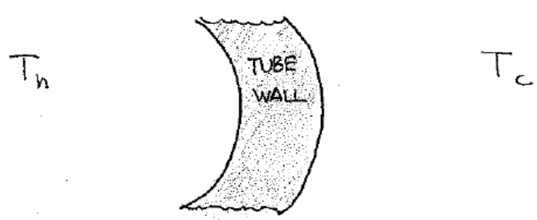
HOW DO WE FIND U ?

FOR A SMALL SECTION of HXR:



$\delta\dot{Q} =$ __________

SIDE VIEW OF INNER TUBE



WHERE

$R_{TOTAL} =$

SO:

$\boxed{UA =}$

* "HXR" IS A COMMON ABBREVIATION FOR HEAT EXCHANGER.

MUST CHOOSE AN AREA ON WHICH TO BASE U:

$$UA = \qquad\qquad =$$

USUALLY BASED ON AREA

ANYWAY...

STILL NEED

$$\dot{Q} = UA\,\Delta T_{AVG} = UA\,\underbrace{(T_h - T_c)_{AVG}}_{=\,?}$$

PROBLEM:

SOLUTION:

ASSUMPTION # 5:

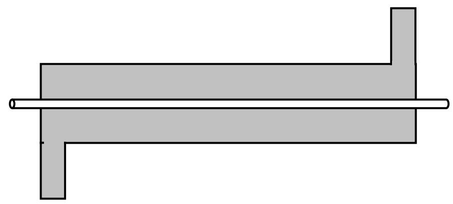
CONSERVATION of ENERGY + ASSUMPTION # 5 YIELDS

$$\dot{Q} = UA \times$$

$$\equiv$$

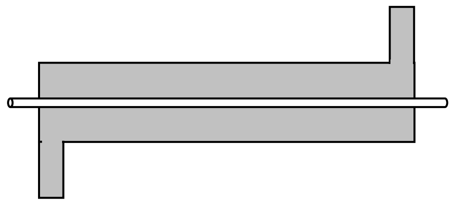
Example

A counter-flow double-pipe heat exchanger is to heat water from 20°C to 80°C at a flow rate of 1.2 kg/s. The warmer fluid is geothermal water available at 160°C and a flow rate of 2 kg/s. The inner tube is thin-walled with a diameter of 1.5 cm. If the **overall heat transfer coefficient** is 640 W/m^2-C°, find the required heat exchanger length.

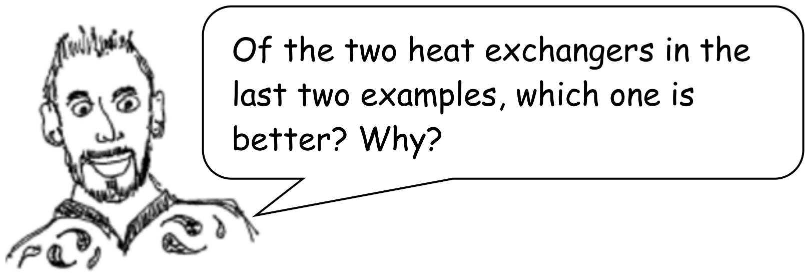


Example

Reconsider the last example, but this time make the heat exchanger a *parallel flow* design. As before, the heat exchanger is a double-pipe design, and is used to heat water from 20°C to 80°C at a flow rate of 1.2 kg/s. The warmer fluid is geothermal water available at 160°C and a flow rate of 2 kg/s. The inner tube is thin-walled with a diameter of 1.5 cm. If the overall heat transfer coefficient is 640 W/m^2-C°, find the required heat exchanger length.

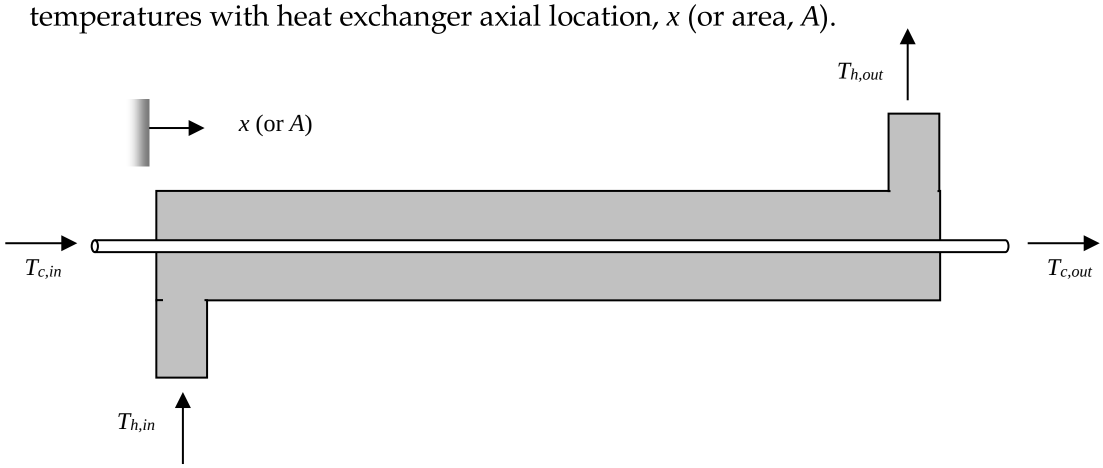


ACTIVE LEARNING EXERCISE—HXR flow directions

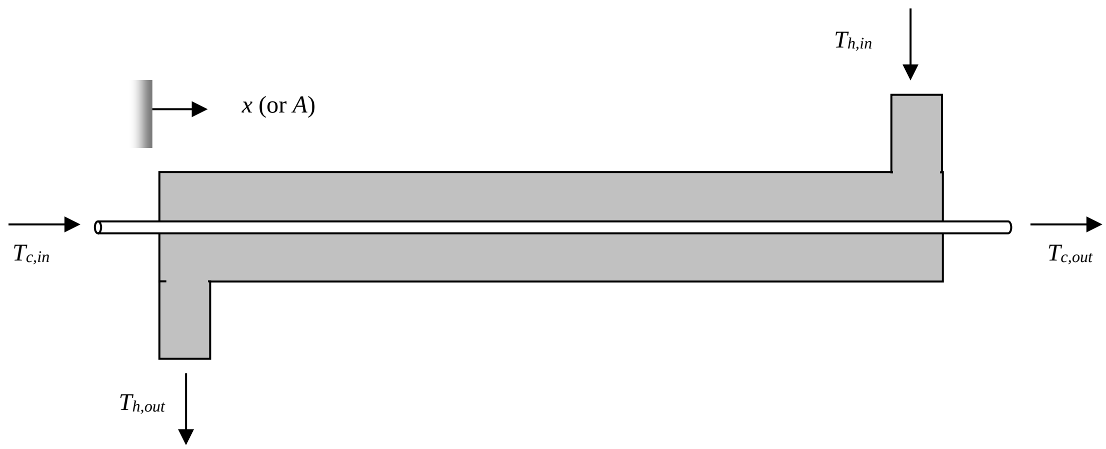


Why is this the case?

Let's explore this a bit more. Consider a *parallel flow* heat exchanger with a warm fluid inlet temperature $T_{h,in}$ and a cold fluid inlet temperature $T_{c,in}$. Sketch the variation of fluid temperatures with heat exchanger axial location, x (or area, A).

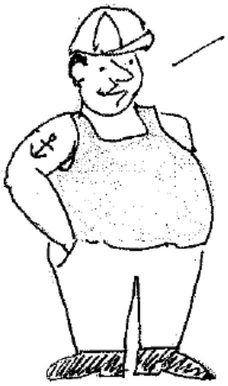


Now consider a *counter-flow* arrangement of the same heat exchanger. The warm fluid inlet temperature is still $T_{h,in}$ and the cold fluid inlet temperature is still $T_{c,in}$. Sketch the variation of fluid temperatures with heat exchanger axial location, x (or area, A).



NOTES: Heat exchangers

TYPES of HEAT EXCHANGERS

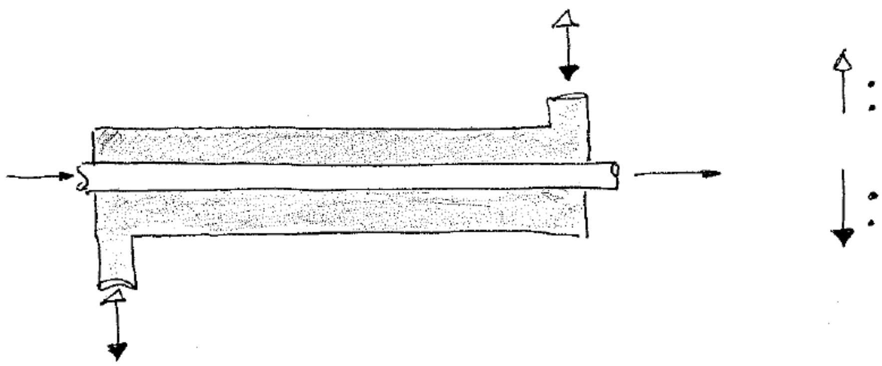


HEAT EXCHANGERS ARE USUALLY CLASSIFIED ACCORDING TO

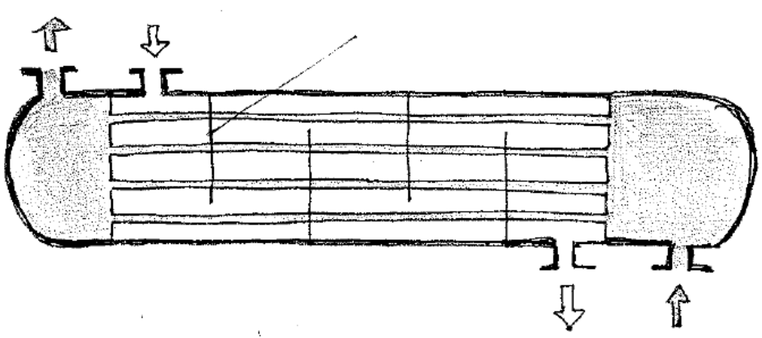
•

•

I.



II.



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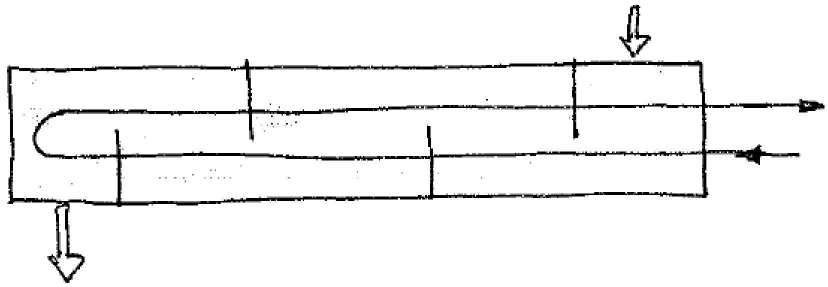
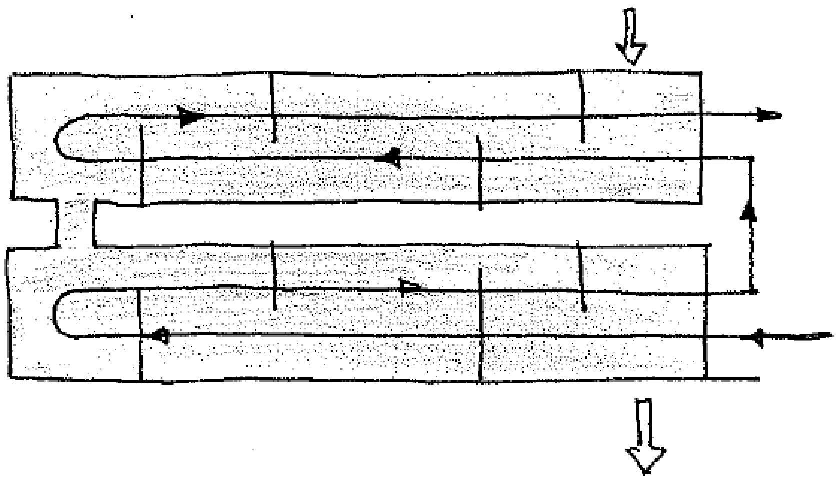
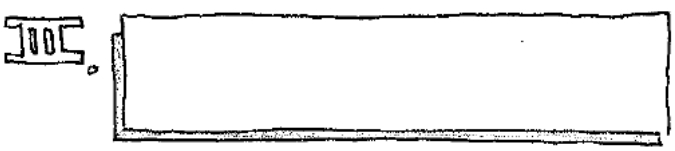
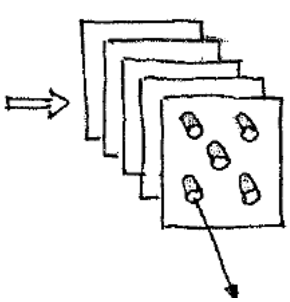
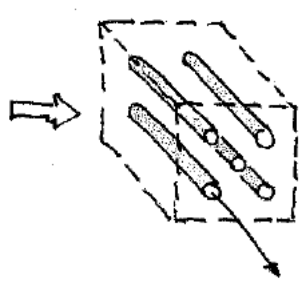
NOTES: Heat exchangers



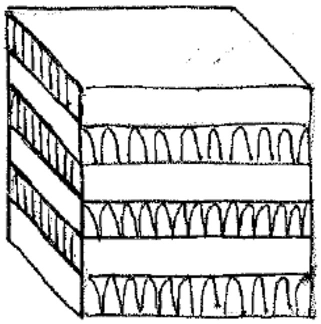


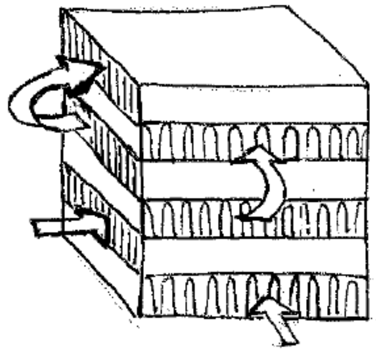






IV.





$\dot{Q} = UA * \Delta T_{LM}$ WAS DERIVED FOR A DOUBLE-PIPE HXR. CAN I USE IT FOR THESE OTHER TYPES?

Answer: YES, IF.......

WHERE

$F = f($ $)$

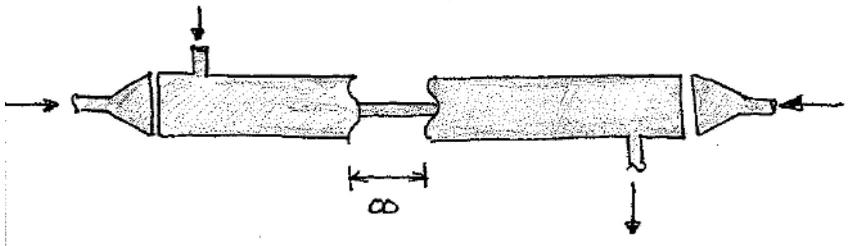
$R =$ ______ *

$P =$ ______ *

* HOT VS. COLD FLUID DOESN'T MATTER HERE

NOTES: Effectiveness-NTU method

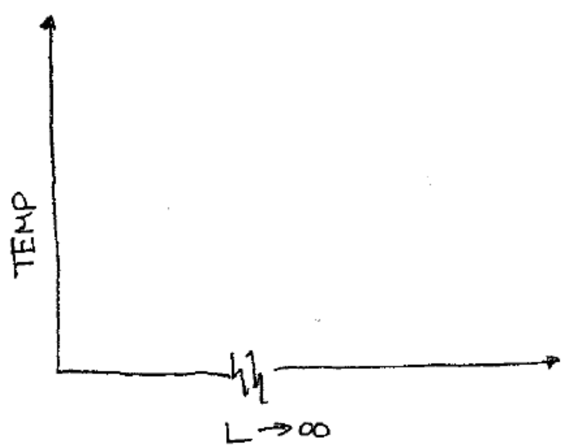
CONSIDER A *Counterflow double-pipe* HXR of ∞ LENGTH:



TWO THINGS CAN HAPPEN.

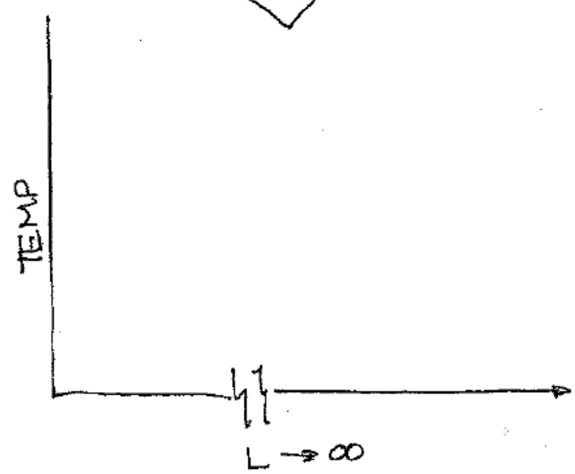
EITHER

The warmer fluid cools all the way down to the cooler fluid inlet temperature



OR

The cooler fluid warms all the way up to the warmer fluid inlet temperature



(HINT: WRITE ENERGY BALANCES FOR BOTH FLUIDS IN TERMS of TEMPERATURE RISE/DROP.)

EITHER WAY $\Delta TEMP_{MAX} =$

THIS LEADS TO A *Maximum heat transfer rate* (WHEN $L \to \infty$)

$$\dot{Q}_{MAX} =$$

AND AN EFFECTIVENESS

$$\varepsilon \equiv \frac{\quad}{\quad} = \frac{\quad}{\quad} \; *$$

$$\stackrel{OR}{=} \frac{\quad}{\quad} \; *$$

* WHICH DO YOU USE?

IT CAN BE SHOWN

$$\varepsilon = f\,(\text{HXR TYPE}, \boxed{\quad}, \boxed{\quad})$$

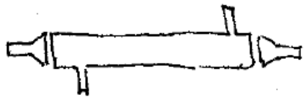
WHERE

$$= \frac{\quad}{\quad} (\qquad)$$

$$= \frac{\quad}{\quad}$$

NOTES: Effectiveness-NTU method

HERE'S WHAT: Say you already have a HXR. You know these things:



- $T_{h,in}$
- $T_{c,in}$
- MASS FLOW RATES
- HXR UA, DIMENSIONS, ETC.

DESCRIBE HOW YOU WOULD USE THE LMTD METHOD TO FIND

- $\dot{Q}$
- $T_{h,out}$
- $T_{c,out}$

NOW USING THE ε-NTU METHOD:

1.)

2.)

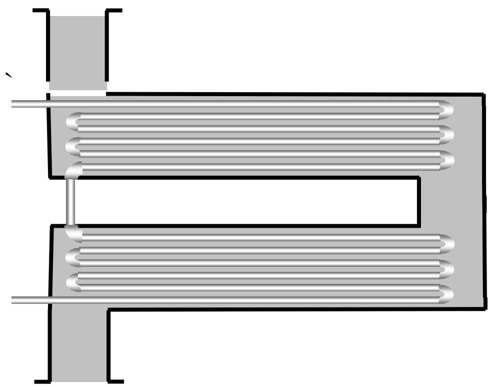
3.)

4.)

Example

0.2 kg/s of hot oil (c_p = 2200 J/kg-°C) is to be cooled by water (c_p = 4180 J/kg-°C) in a 2-12 shell and tube HXR. The water flows through thin-walled tubes with a diameter of 1.8 cm at a rate of 0.1 kg/s. The length of each tube pass is 3 m and the overall heat transfer coefficient is 340 W/m^2-°C. (Tube side or shell side? Does it matter?) The inlet temperatures of the oil and water are 160°C and 18°C, respectively.

(a) Find the rate of heat transfer in the exchanger and
(b) the exit temperatures of both fluids.



ACTIVE LEARNING EXERCISE: ε-NTU Discovery Session

The effectiveness-NTU (*ε-NTU*) method not only gives us an easy way to perform heat exchanger *analysis* problems, it gives us physical insight into the performance of HXRs. The basis of this insight is that the *effectiveness*, ε, tells us how well our HXR performs compared to the theoretically best heat exchanger. Using the *ε-NTU* relationships (equations and charts), answer the following questions.

(1) What is the possible range for effectiveness? (Holy cow, that's easy!)

(2) For a given *NTU* and *C*, which heat exchanger construction/flow direction combination has the highest effectiveness?

(3) How does effectiveness vary with *C*?

(4) For what value of *C* is effectiveness at its maximum?

a. How does this value of *C* for ε_{max} vary with HXR type? flow direction?

b. For this value of C, what does this mean for one of the fluid's $m_{dot}c_p$ value? What does it mean about this fluid *physically*?

(5) If $NTU < 0.3$, which equation would you use for ε? Why?

(6) Let's say you are thinking about increasing the effectiveness of your HXR by increasing its UA value. You can do this in two ways:

a. You can increase flowrate(s) which increases h(s) and thereby U. But that means increasing your operational cost. (Bigger Δp means bigger pumping power required.)

b. You can increase A, but that increases the capital cost of the HXR. (Bigger A means more material to build the HXR.)

By consulting the ε -NTU charts, come up with a criterion by which you can determine whether it is worth the increase in either operational or capital cost to increase your UA. (Hint: Think about where UA shows up in the ε-NTU method.)

BOILING HEAT TRANSFER

- BOILING OCCURS AT A SOLID – LIQUID INTERFACE WHEN THE TEMPERATURE of THE SOLID, T_s, IS SUFFICENTLY ABOVE THE SATURATION TEMPERTURE of THE LIQUID, T_{SAT}.
- THE DIFFERENCE BETWEEN THE SURFACE & SATURATION TEMPERATURES IS KNOWN AS THE ________________.

BOILING IS CONSIDERED A FORM of CONVECTION, & <u>BOILING HEAT FLUX</u> IS EXPRESSED AS

$$\boxed{\dot{q}_{BOILING} = h\,(T_s - T_{SAT}) = \rule{5cm}{0.4pt}}\quad (W/m^2)$$

SINGLE-PHASE CONVECTION DEPENDS ON MANY PROPERTIES SUCH AS ρ, μ, k, c_p, *etc.* BOILING ALSO DEPEND ON THESE, *<u>FOR BOTH PHASES</u>*, AS WELL AS

h_{fg}: ________________ &

σ: ________________.

- DEPENDING ON THE STATE of *<u>BULK</u>* MOTION of THE FLUID, BOILING CAN BE CLASSIFIED AS

________ OR ________ BOILING.
(1) (2)

- BOILING CAN ALSO BE CLASSIFIED BASED ON THE BULK LIQUID TEMPERATURE. IN THE CASE WHERE THE BULK LIQUID TEMPERATURE IS

  1) LESS THAN T_{SAT}, WE HAVE ______________________.

  2) IF $T_{BULK,LIQUID} \approx T_{SAT}$, WE HAVE ______________________.

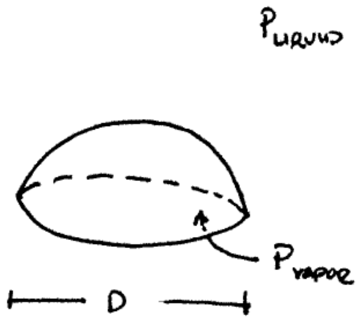
- IN ADDITION TO THE INHERENT COMPLEXITY OF CONVECTION (NATURAL &/OR FORCED) & PHASE CHANGE, BOILING IS FURTHER COMPLICATED BY

  THERMODYNAMIC NON-EQUILIBRIUM.

  IN PARTICULAR, __________ ______________ ARE GENERALLY NOT IN THERMODYNAMIC EQUILIBRIUM WITH THE __________ __________.

CONSIDER A VAPOR BUBBLE:

(CUT IN HALF)

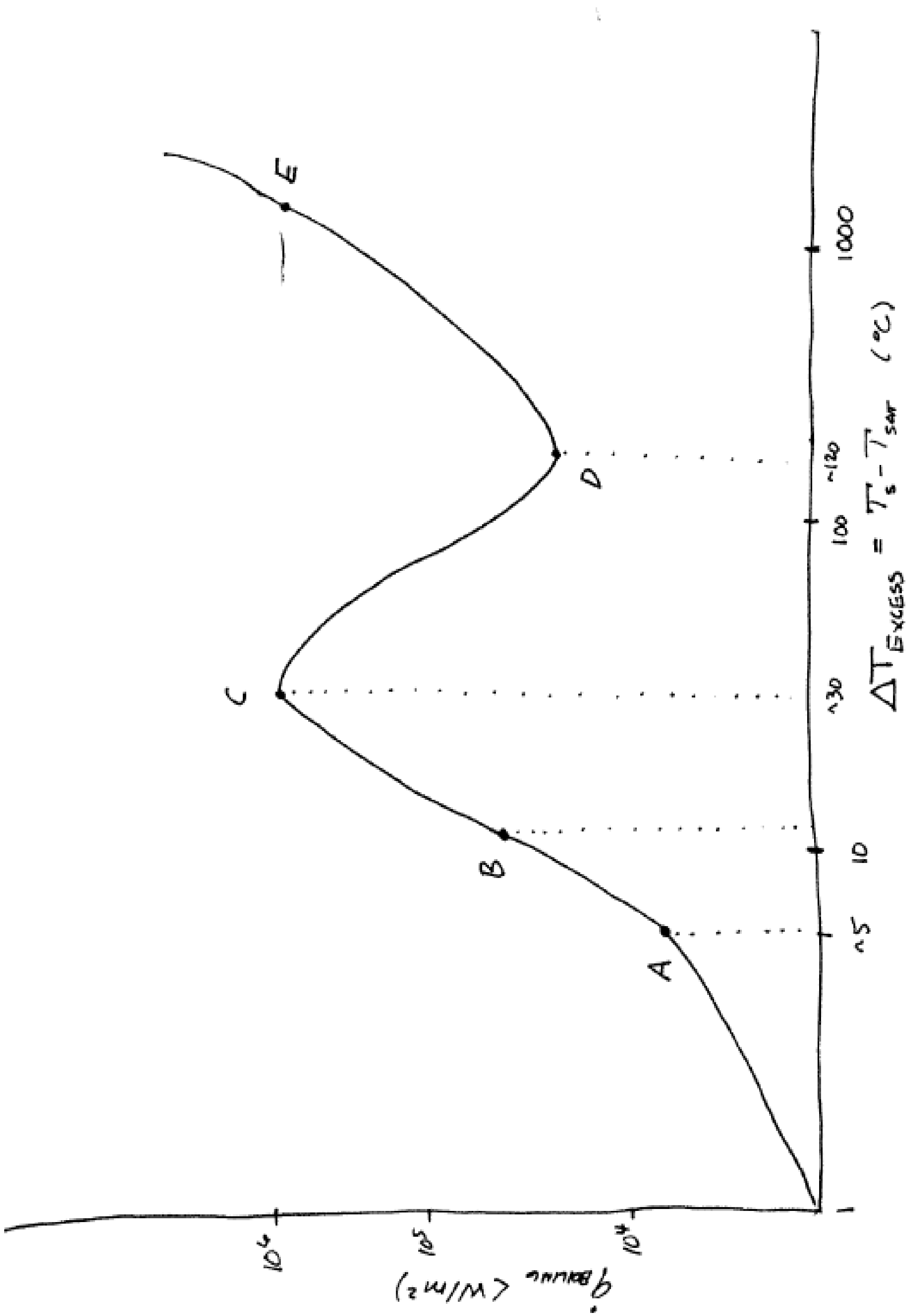


Find: RELN BETWEEN P_V, P_L & σ.

Soln:

FORCE BALANCE ON THE BUBBLE:

NOTES: Boiling heat transfer



POOL BOILING

BOILING REGIMES & THE BOILING CURVE:

A FUNCTIONAL DEPENDANCE EXISTS BETWEEN BOILING HEAT FLUX & EXCESS TEMPERATURE. THIS DEPENDENCE IS ILLUSTRATED ON THE __________ __________.

THE BOILING CURVE IS DIVIDED INTO A NUMBER OF REGIMES.

1) NATURAL CONVECTION BOILING (WHERE IS IT ON THE CURVE?)
(WHAT ARE SOME CHARACTERISTICS OF THIS REGIME?)

2) NUCLEATE BOILING (WHERE IS IT ON THE CURVE?)

3) TRANSITION BOILING

4) FILM BOILING

CRITICAL HEAT FLUX

- IN HEAT INPUT CONTROLLED SITUATIONS (MOST REAL SITUATIONS) THE BOILING CURVE BETWEEN ____ & _____ IS BY-PASSED ALMOST INSTANTANEOUSLY, RESULTING IN SURFACE TEMPERATURES ON THE ORDER of 1000°C. FOR THIS REASON, CRITICAL HEAT FLUX (CHF) IS ALSO KNOWN AS

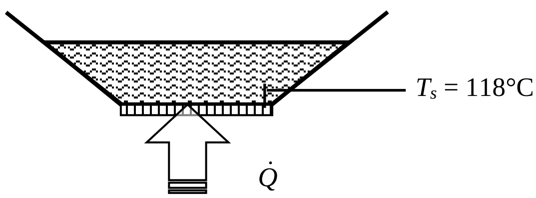
THE ____________ ____________ OR SIMPLY

____________.

Example

A starving Rose-Hulman student is preparing Ramen Noodles in a copper-bottomed pan bought from Goodwill. The diameter of the bottom of the pan is 0.3-m, and is maintained at 118°C by an electric heating element.

(a) Estimate the power required to boil the water in the pan.
(b) What is the evaporation rate?
(c) Estimate the critical heat flux.
(d) Estimate the number of shrimp used to create one flavor packet for shrimp-flavored Ramen Noodles.



Cartoon summaries, charts, tables, and other miscellaneous resources

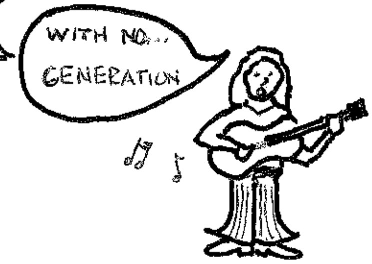
Forms of the conduction equation

Conduction equation	1-D or 3-D?	Coordinate system?	Constant properties?
$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) + \dot{e}_{gen}$$			
$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 T}{\partial \phi^2} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(r \sin\theta \frac{\partial T}{\partial \theta}\right) + \frac{\dot{e}_{gen}}{k}$$			
$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{\dot{e}_{gen}}{k}$$			
$$\rho c \frac{\partial T}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(k r^2 \frac{\partial T}{\partial r}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial}{\partial \phi}\left(k \frac{\partial T}{\partial \phi}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(k r \sin\theta \frac{\partial T}{\partial \theta}\right) + \dot{e}_{gen}$$			
$$\rho c \frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(k r \frac{\partial T}{\partial r}\right) + \dot{e}_{gen}$$			
$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{e}_{gen}}{k}$$			

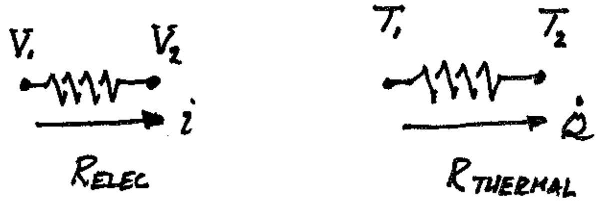
Forms of the conduction equation

Conduction equation	1-D or 3-D?	Coordinate system?	Constant properties?
$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\dot{e}_{gen}}{k}$			
$\rho c\frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{e}_{gen}$			
$\rho c\frac{\partial T}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right) + \dot{e}_{gen}$			
$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{e}_{gen}}{k}$			
$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{\dot{e}_{gen}}{k}$			
$\rho c\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{e}_{gen}$			

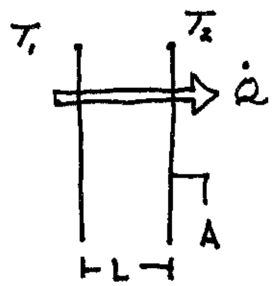
STEADY-STATE, 1-D CONDUCTION



THE RESISTANCE ANALOGY

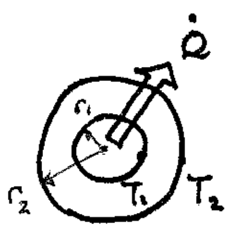


PLANE WALL



$$R_{THERM} = \frac{L}{kA}$$

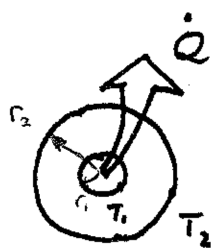
CYLINDER



$$R_{THERM} = \frac{\ln(r_2/r_1)}{2\pi L k}$$

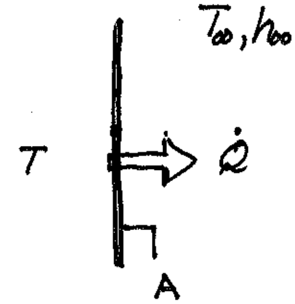
LENGTH INTO PAGE

SPHERE



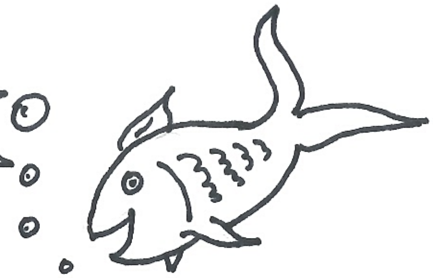
$$R_{SPH} = \frac{r_2 - r_1}{4\pi r_1 r_2 k}$$

CONVECTION (ANY GEOMETRY)



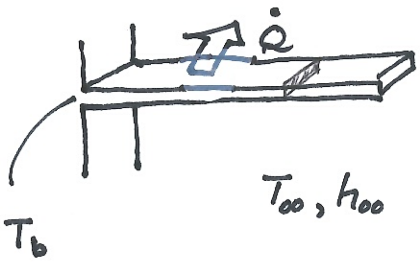
$$R_{THERM} = \frac{1}{hA}$$

FINS



TO INCREASE $\dot{Q}_{conv}$

- INCREASE h -or-
- INCREASE $\underline{\underline{A}}$ (I.E. USE A FIN)



P = PERIMETER

A_c = CROSS SECTIONAL AREA

k = CONDUCTIVITY of FIN MATERIAL

FOR OOLY LONG FINS

$$\dot{Q} = \sqrt{hPkA_c}\,(T_b - T_\infty)$$

FOR INSULATED TIP:

$$\dot{Q} = \sqrt{hPkA_c}\,\tanh\left[\left(\frac{hP}{kA_c}\right)^{1/2} L\right](T_b - T_\infty)$$

FOR CONST. CROSS SECTIONAL AREA

IN GENERAL

$$\boxed{\dot{Q}_{FIN} = \eta_{FIN}\,\dot{Q}_{MAX}}$$

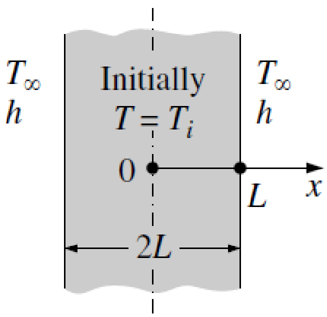
FIN EFFICIENCY (FROM CHARTS. FUNCTION of GEOM, B.C, ETC.)

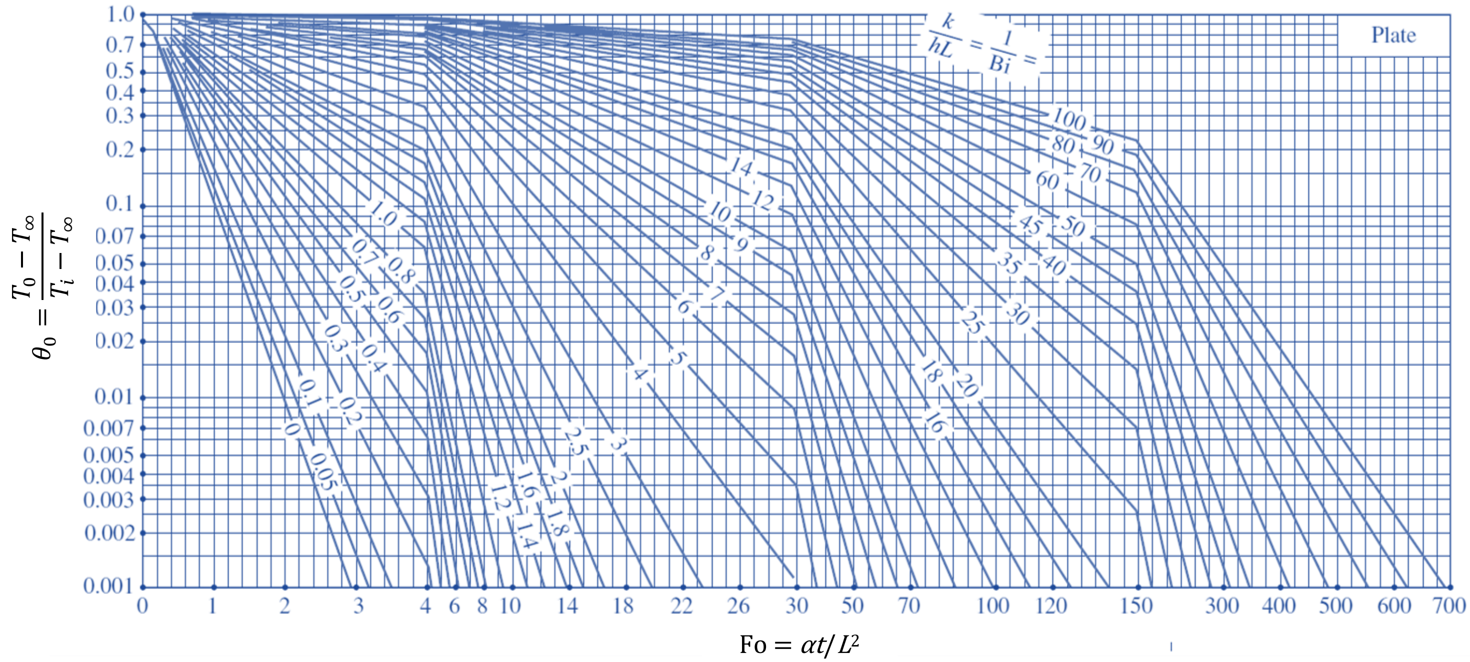
$\dot{Q}_{MAX} = h A_{FIN} (T_b - T_\infty)$ (HEAT TRANSFER FROM FIN IF ENTIRE FIN WERE AT T_b)

FIN·EFFECTIVENESS

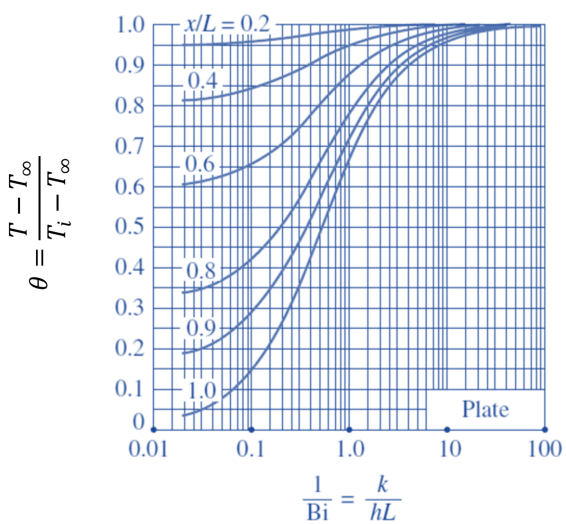
$$\epsilon = \frac{\dot{Q}_{W/FIN}}{\dot{Q}_{W/NO\ FIN}} = \frac{\dot{Q}_{FIN} + \dot{Q}_{UNFINNED}}{\dot{Q}_{W/\ NO\ FIN}}$$

1st term solutions for 1-D transient conduction in an infinite plane

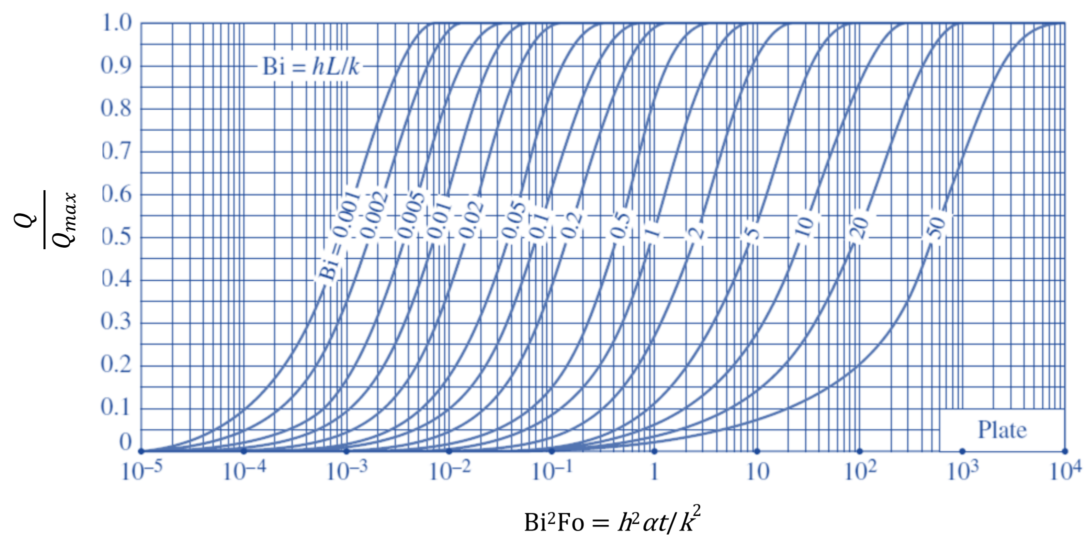




(a) Midplane temperature

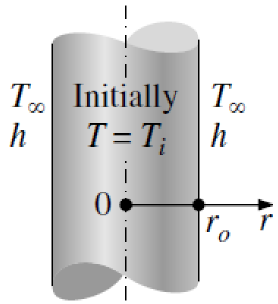


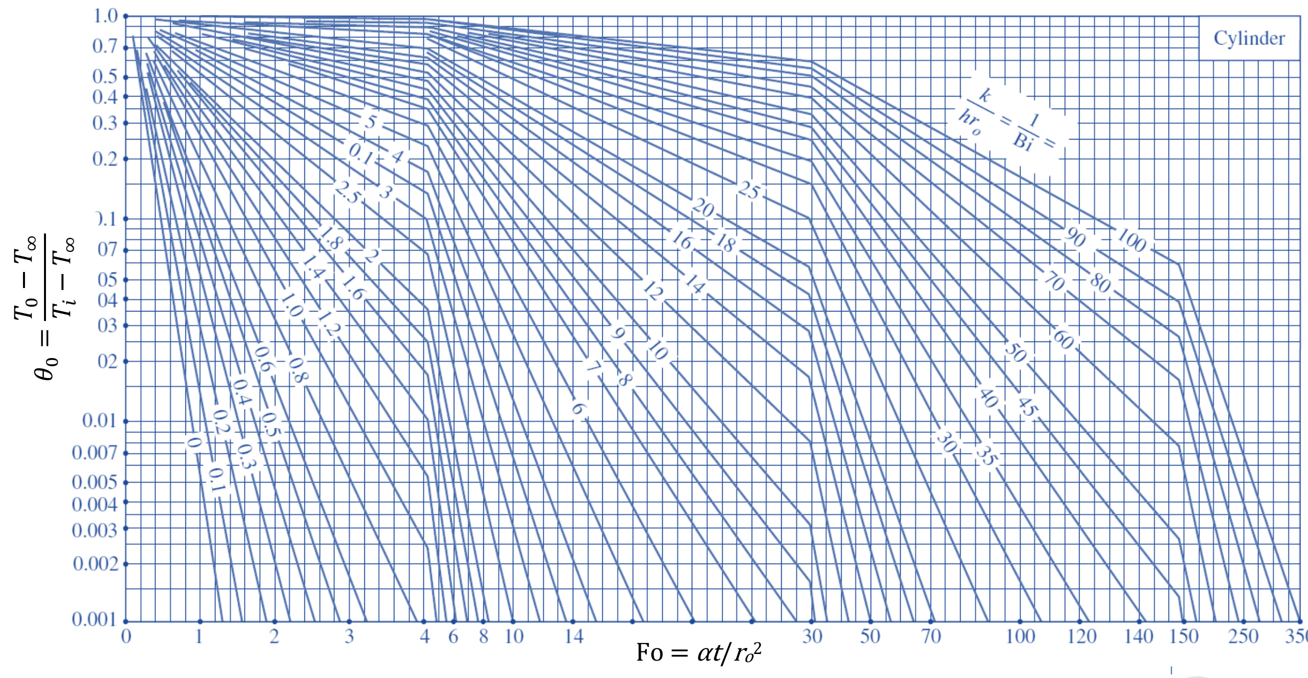
(b) Temperature



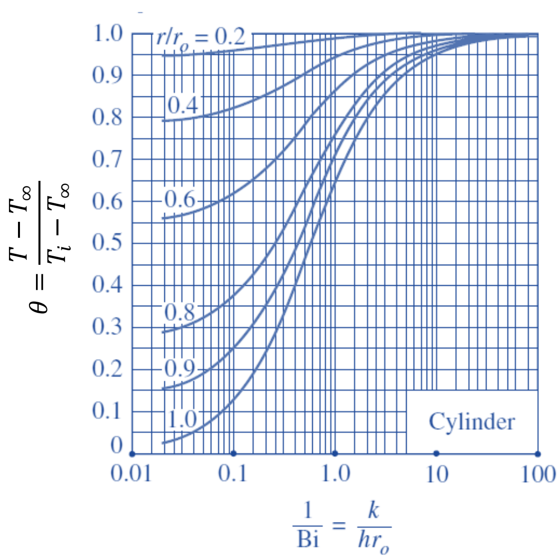
(c) Total heat transfer

1st term solutions for 1-D transient conduction in an infinite cylinder

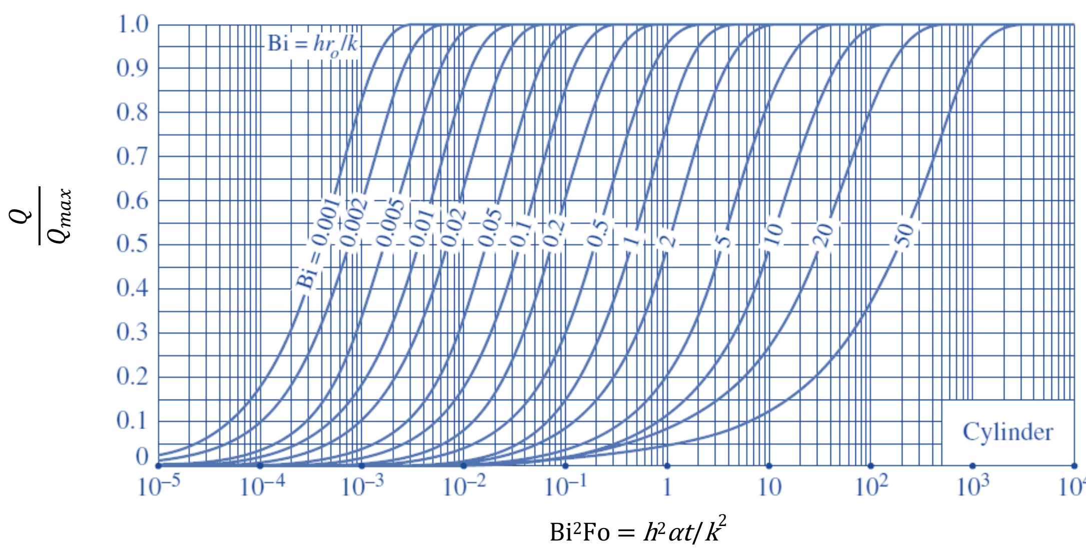




(a) Midplane temperature

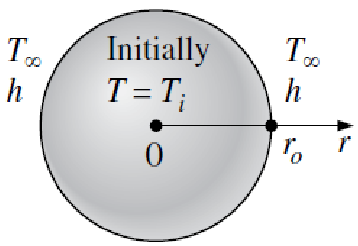


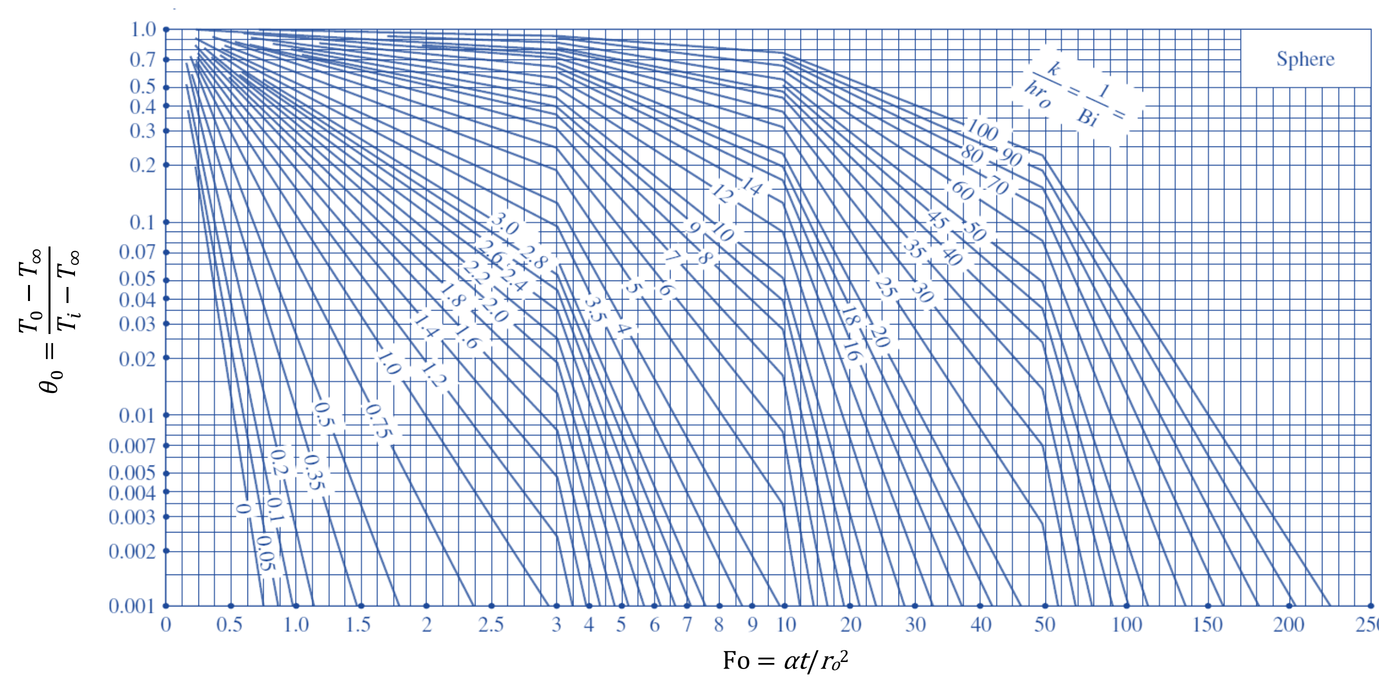
(b) Temperature



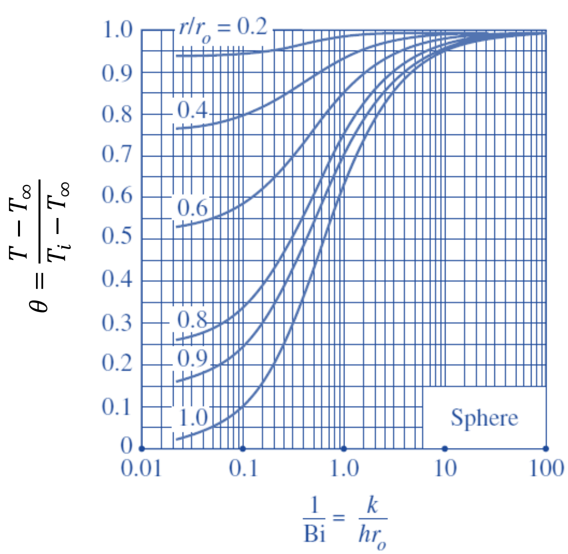
(c) Total heat transfer

1st term solutions for 1-D transient conduction in a sphere

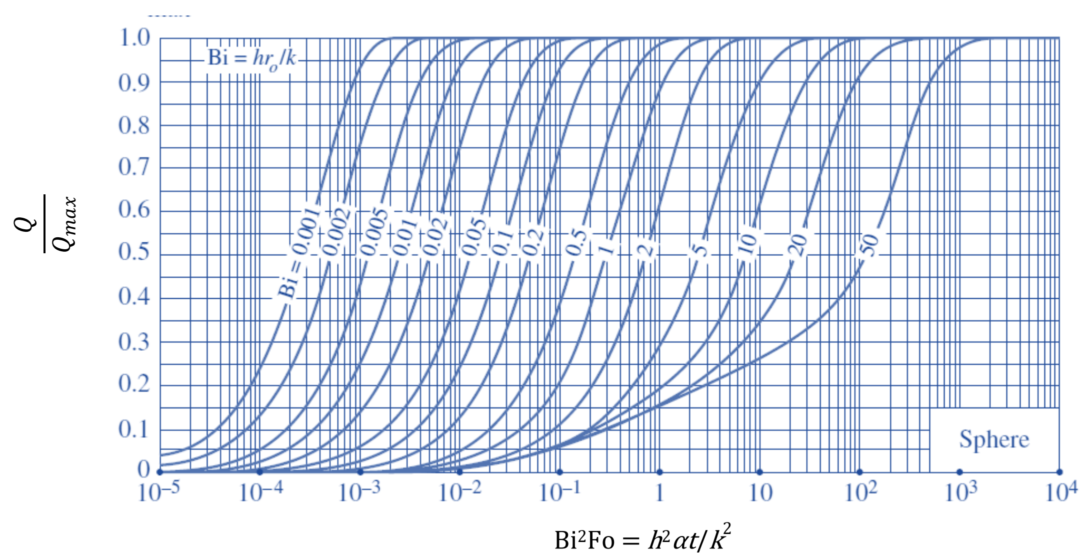




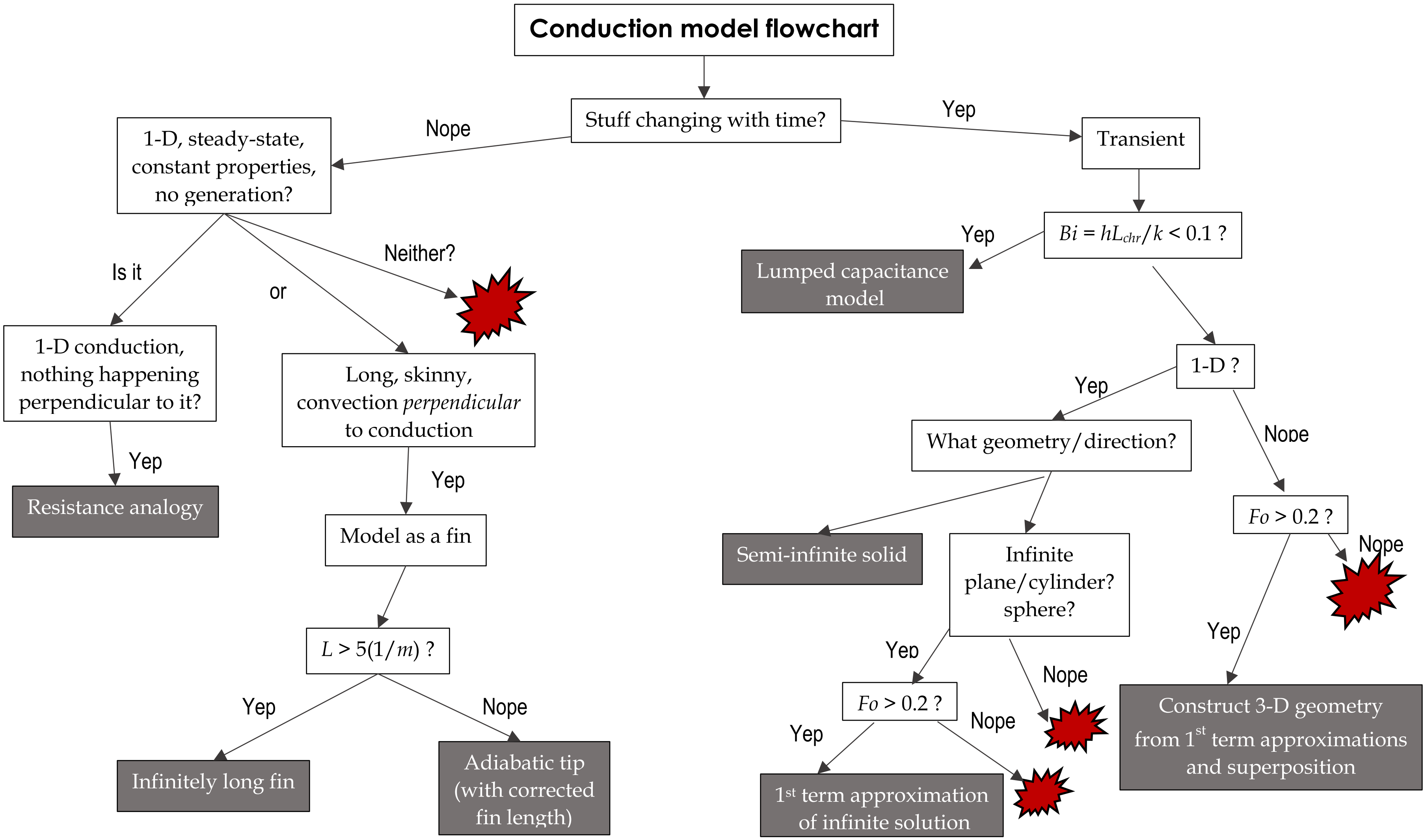
(a) Midplane temperature



(b) Temperature



(c) Total heat transfer



Conduction model flowchart
Stuff changing with time?
Nope
Yep
1-D, steady-state, constant properties, no generation?
Transient
Is it
or
Neither?
1-D conduction, nothing happening perpendicular to it?
Long, skinny, convection *perpendicular* to conduction
Yep
Resistance analogy
Yep
Model as a fin
$L > 5(1/m)$?
Yep
Nope
Infinitely long fin
Adiabatic tip (with corrected fin length)
$Bi = hL_{chr}/k < 0.1$?
Yep
Lumped capacitance model
1-D ?
Yep
What geometry/direction?
Nope
Semi-infinite solid
Infinite plane/cylinder? sphere?
$Fo > 0.2$?
Nope
Yep
Yep
Nope
$Fo > 0.2$?
Yep
Nope
1st term approximation of infinite solution
Construct 3-D geometry from 1st term approximations and superposition

TRANSIENT HEAT CONDUCTION

THE LUMPED CAPACITANCE METHOD

BIG ASSUMPTION : $T \neq f(x,y,z)$

$T = f$ (TIME ONLY)

$$\boxed{\frac{T(t) - T_\infty}{T_{initial} - T_\infty} = e^{-\frac{hA}{\rho \forall c} t} = e^{-t/t_c}}$$

WHERE:

$$t_c = \frac{\rho \forall c}{hA}$$

= TIME CONSTANT

(NO DOT!)

MAX Q OCCURS WHEN $t \rightarrow \infty$

$$\boxed{Q_{MAX,IN} = mc(T_\infty - T_i)}$$

VALID WHEN

$$\mathrm{Bi} \equiv \frac{hL_c}{k} < 0.1$$

= <u>BIOT NUMBER</u>

$$L_c \equiv \frac{\forall}{A}$$

= <u>CHR. LENGTH</u>

WHAT HAPPENS IF $\mathrm{Bi} > 0.1$?   *READ ON..........*

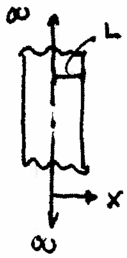
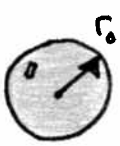
1-D TRANSIENT CONDUCTION

TAKE A THING & PUT IT IN A MEDIUM @ T_∞ W/ h KNOWN.

h, T_∞ **THING** h, T_∞

THING IS *INITIALLY* @ $T_{INITIAL}$ EVERYWHERE. (UNIFORM T)

NOW, FOR 1D TRANSIENT CONDUCTION...

THING		$\theta = \dfrac{T(x \text{ or } r, t) - T_\infty}{T_{initial} - T_\infty}$	Q/Q_{MAX}
	PLANE WALL	$A_1 e^{-\lambda_1^2 F_o} \times \cos\left(\dfrac{\lambda_1 x}{L}\right)$	$1 - \theta_o \dfrac{\sin\lambda_1}{\lambda_1}$
	INFINITE CYLINDER	$A_1 e^{-\lambda_1^2 F_o} \times J_o\left(\dfrac{\lambda_1 r}{r_o}\right)$	$1 - 2\theta_o \dfrac{J_1(\lambda_1)}{\lambda_1}$
	SPHERE	$A_1 e^{-\lambda_1^2 F_o} \times \dfrac{\sin\left(\frac{\lambda_1 r}{r_o}\right)}{\lambda_1 r/r_o}$	$1 - 3\theta_o \dfrac{\sin\lambda_1 - \lambda_1\cos\lambda_1}{\lambda_1^3}$

WHERE: A_1 & λ_1 ARE $f\left(Bi = \dfrac{h(L \text{ or } r_o)}{k}\right)$  CAREFUL!!

F_o = FOURIER NUMBER $\equiv \dfrac{\alpha t}{(L \text{ or } r_o)^2}$

$\theta_o = \theta$ @ (x or $r = 0$)

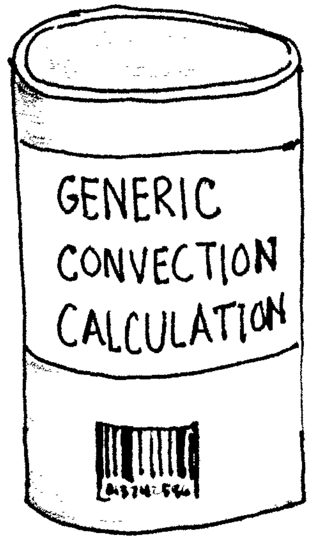
J_o = BESSEL FUNCTION (ZEROTH ORDER)

J_1 = " " (1ST ORDER)

$Q_{MAX} = mc(T_i - T_\infty)$

TMA

HOW TO PERFORM A



1. BECOME AWARE of THE GEOMETRY. IS IT A FLAT PLATE? A CYLINDER?

2. SPECIFY THE APPROPRIATE REFERENCE TEMPERATURE & FIND THE FLUID PROPERTIES.

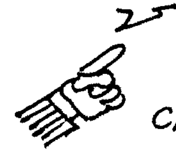
USUALLY (NOT ALWAYS) YOU WANT THE

FILM TEMPERATURE:

$$\boxed{T_f \equiv \frac{T_s + T_\infty}{2}}$$

3. CALCULATE THE REYNOLD'S NUMBER

$$\boxed{Re \equiv \frac{\rho V (L \text{ or } D \text{ etc.})}{\mu}} = \frac{V (L, D \text{ etc.})}{\nu}$$



CAREFUL!

4. DECIDE IF YOU WANT THE LOCAL OR AVERAGE HEAT TRANSER COEFFICIENT.

5. SELECT THE APPROPRIATE NUSSELT CORRELATION.

(REMEMBER $Nu \equiv \dfrac{h (L, D \text{ etc.})}{k_{FLUID}}$.)

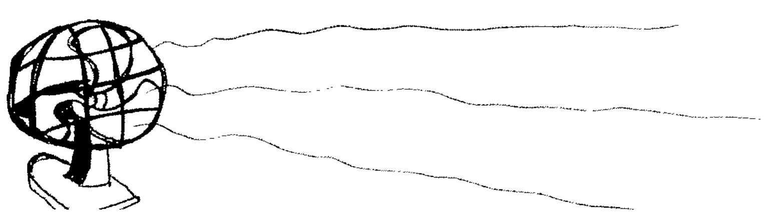
SUMMARY of CORRELATIONS (FOR EXTERNAL FLOW)

Correlations for T_S = const. Boundary Condition

Correlation	Geometry	Conditions
$C_{f,x} = 0.664 Re_x^{-1/2}$	Flat plate	Laminar, Local, Use T_f
$Nu_x = 0.332 Re_x^{1/2} Pr^{1/3}$	Flat plate	Laminar, Local, Use T_f, $Pr > 0.6$
$C_f = 1.328 Re_L^{-1/2}$	Flat plate	Laminar, Average, Use T_f
$Nu = 0.664 Re_L^{1/2} Pr^{1/3}$	Flat plate	Laminar, Average, Use T_f, $0.6 < Pr < 50$
$C_{f,x} = 0.0592 Re_x^{-1/5}$	Flat plate	Turbulent, Local, Use T_f, $5 \times 10^5 < Re_x < 10^7$
$Nu_x = 0.0296 Re_x^{4/5} Pr^{1/3}$	Flat plate	Turbulent, Local, Use T_f, $5 \times 10^5 < Re_x < 10^7$, $Pr > 0.6$
$C_f = 0.074 Re_L^{-1/5}$	Flat plate	Turbulent, Average, Use T_f, $5 \times 10^5 < Re_x < 10^7$
$Nu = 0.037 Re_L^{4/5} Pr^{1/3}$	Flat plate	Turbulent, Average, Use T_f, $5 \times 10^5 < Re_x < 10^7$, $Pr > 0.6$
$C_f = 0.074 Re_L^{-1/5} - 1742 Re_L^{-1}$	Flat plate	Mixed laminar and turbulent flow, Average, Use T_f, $5 \times 10^5 < Re_x < 10^7$
$Nu = (0.037 Re_L^{4/5} - 871) Pr^{1/3}$	Flat plate	Mixed laminar and turbulent flow, Average, Use T_f, $5 \times 10^5 < Re_x < 10^7$, $0.6 < Pr < 60$
$Nu_D = 0.3 + \dfrac{0.62 Re_D^{1/2} Pr^{1/3}}{[1 + (0.4/Pr)^{2/3}]^{1/4}} \left[1 + \left(\dfrac{Re_D}{282{,}000}\right)^{5/8}\right]^{4/5}$	Circular cylinder	Average, Use T_f, $Re_D Pr > 0.2$
$Nu_D = 2 + [0.4 Re_D^{1/2} + 0.06 Re_D^{2/3}] Pr^{0.4} \left(\dfrac{\mu_\infty}{\mu_S}\right)^{1/4}$	Sphere	Average, Use T_∞ for all properties except μ_S, for which you use T_S, $3.5 < Re < 80{,}000$, $0.7 < Pr < 380$
$Nu = C Re^m Pr^n$	Circular and noncircular cylinders	Average, Use T_f, Use Tables in text to find C, m and n and Re ranges.

Correlations for $\dot{q}$ = const. Boundary Condition

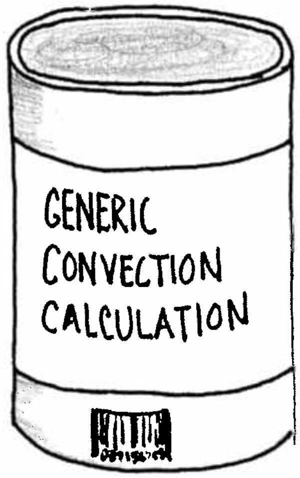
Correlation	Geometry	Conditions
$Nu_x = 0.453 Re_x^{1/2} Pr^{1/3}$	Flat plate	Laminar, Local, Use T_f, $Pr > 0.6$
$Nu = 0.906 Re_L^{1/2} Pr^{1/3}$	Flat plate	Laminar, Average, Use T_f, $0.6 < Pr < 50$
$Nu_x = 0.0308 Re_x^{4/5} Pr^{1/3}$	Flat plate	Turbulent, Local, Use T_f, $5 \times 10^5 < Re_x < 10^7$, $Pr > 0.6$
$Nu_x = 0.0385 Re_x^{4/5} Pr^{1/3}$	Flat plate	Turbulent, Average, Use T_f, $5 \times 10^5 < Re_x < 10^7$, $Pr > 0.6$



TMA

HOW TO PERFORM A →

II - *Internal Flow*



1. BECOME AWARE of THE <u>GEOMETRY</u>. IF ITS A *NON-CIRCULAR DUCT*, FIND

$$\boxed{D_h \equiv \frac{4A_c}{P}}$$

THE HYDRAULIC DIAMETER

2. SPECIFY THE APPROPRIATE <u>REFERENCE TEMPERATURE</u> &

FIND THE FLUID PROPERTIES. <u>USUALLY</u> (NOT ALWAYS) YOU WANT THE

<u>BULK MEAN FLUID TEMPERATURE</u>

$$\boxed{T_b \equiv \frac{T_{m,in} + T_{m,out}}{2}}$$

3. CALCULATE THE <u>REYNOLD'S NUMBER</u>

$$\boxed{Re \equiv \frac{\rho V (D \text{ or } D_h)}{\mu}} = \frac{V(D \text{ or } D_h)}{\nu}$$

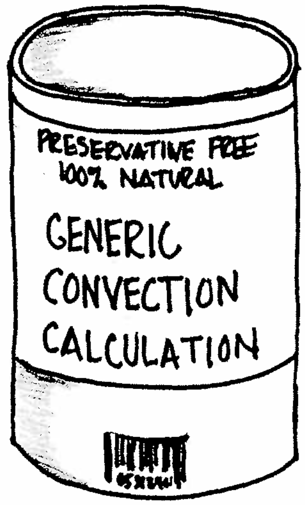
CAREFUL!

& DETERMINE IF THE FLOW IS

<u>FULLY-DEVELOPED</u> -or- <u>DEVELOPING</u>

HOW TO PERFORM A

III - *Natural Convection*



1. BECOME AWARE of THE GEOMETRY.

2. SPECIFY THE APPROPRIATE REFERENCE TEMPERATURE & FIND THE PROPERTIES

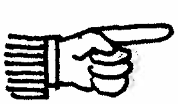
USUALLY THE FILM TEMPERATURE

$$\boxed{T_f = \frac{T_s + T_\infty}{2}}$$

3. CALCULATE THE GRASHOF &/OR RAYLEIGH NUMBER(s)

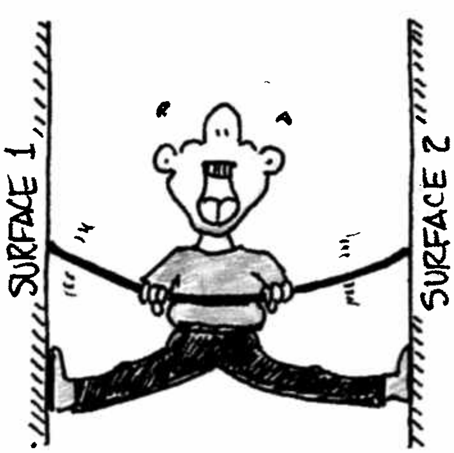
$$\boxed{Gr \equiv \frac{g\beta(T_s - T_\infty)\,\delta^3}{\nu^2} \qquad Ra \equiv Gr * Pr}$$

4. SELECT THE APPROPRIATE NUSSELT CORRELATION.



ASSUMED B.C. ON MOST CORRELATIONS IS $T_s = \text{CONST.}$

¿ What to do if q = constant ?



ENCLOSURES

DO ABOVE 4 STEPS W/ THESE CHANGES ⇨

2E. USE $T_{AV} = \frac{T_{s,1} + T_{s,2}}{2}$

3E. USE $(T_{s,1} - T_{s,2})$ TO FIND Gr &/OR Ra.

... THEN ...

5E. FIND $\boxed{k_{EFF} = k_{FLUID}\, Nu}$ & THEN TREAT THE ENCLOSED SPACE AS A SOLID SUBJECT TO S-S, 1-D CONDUCTION (w/ $k_{SOLID} = k_{EFF}$)

COMBINED

FORCED & NATURAL CONVECTION

IF	THEN
$\frac{Gr}{Re^2} < 0.1$	ONLY FORCED CONVECTION IS IMPORTANT
$\frac{Gr}{Re^2} > 10$	ONLY NATURAL CONVECTION IS IMPORTANT
$0.1 < \frac{Gr}{Re^2} < 10$	*BOTH* ARE IMPORTANT AND YOU NEED ⇨

$$\boxed{Nu_{COMBINED} = \left(Nu_{FC}^{n} \pm Nu_{NC}^{n}\right)^{1/n}}$$

FORCED *ONLY*

+ IF NC "HELPS"
− IF NC "HURTS"

NATURAL *ONLY*

TMA

4. DETERMINE THE BOUNDARY CONDITION. IF ITS T_s = CONSTANT (& YOU WANT THE *AVERAGE* h) YOU NEED THE

LOG MEAN TEMPERATURE DIFFERENCE

$$\boxed{\Delta T_{LM} \equiv \frac{(T_s - T_{m,out}) - (T_s - T_{m,in})}{\ln \dfrac{T_s - T_{m,out}}{T_s - T_{m,in}}}}$$

5. SELECT THE APPROPRIATE CORRELATION...

SUMMARY of CORRELATIONS

(FOR INTERNAL FLOW)

Correlations for T_S = const. Boundary Condition

Correlation	Geometry	Conditions
$f = 64/Re_D$	Circular duct	Laminar, Fully developed, Use T_b
$Nu_D = 3.66$	Circular duct	Laminar, Fully developed, Use T_b
$Nu = 1.86\left(\dfrac{RePrD}{L}\right)^{1/3}\left(\dfrac{\mu}{\mu}\right)^{0.14}$	Circular duct	Laminar, Developing, Use T_b for all properties except μ_S, for which you use T_S
f = constant/Re_{Dh}	Non-circular duct	Laminar, Fully developed, Use T_b, Use Tables in text to find constant
Nu_{Dh} = constant	Non-circular duct	Laminar, Fully developed, Use T_b, Use Tables in text to find constant
$f = 0.184 Re_{Dh}^{-0.2}$	Circular or non-circular ducts	Turbulent, Fully developed, **smooth surfaces**, Use T_b
$f \Rightarrow$ Use Moody Chart	Circular or non-circular ducts	Turbulent, Fully developed, smooth or rough surfaces, Use T_b
$Nu_{Dh} = 0.125 * f * Re_{Dh} * Pr^{1/3}$	Circular or non-circular ducts	Turbulent, Fully developed, smooth or rough surfaces, Use T_b
$Nu_{Dh} = 0.023 * Re_{Dh}^{0.8} * Pr^n$ n = 0.4 for heating, = 0.3 for cooling	Circular or non-circular ducts	Turbulent, Fully developed, smooth or rough surfaces, Use T_b, $0.7 < Pr < 160$, $Re >$ 10,000

(TMA)

Correlations for $\dot{q}$ = const. Boundary Condition

Correlation	Geometry	Conditions
$Nu_D = 4.36$	Circular duct	Laminar, Fully developed, Use T_b
Nu_{Dh} = constant	Non-circular duct	Laminar, Fully developed, Use T_b, Use Tables in text to find constant

Turbulent flow is rather insensitive to boundary conditions. Use previous correlations.