Example

Consider a heat exchanger and a steam turbine used as a waste heat recovery system. The heat exchanger takes hot combustion gases and uses them to heat steam, which in turn passes through a turbine. The gases can be modeled as air treated as an ideal gas with variable specific heats. The surroundings are at $T_0 = 25^\circ$C and $P_0 = 101$ kPa.

(a) Find the power (in kW) delivered by the turbine.
(b) Find the isentropic (adiabatic efficiency) of the turbine.
(c) For the heat recovery system (heat exchanger and turbine combined) identify
   1. where inflows of exergy occur
      \[ \text{W/AIR IN}, \text{ W/STEAM IN} \]
   2. outflows of exergy occur
      \[ \text{W/AIR OUT}, \text{ W/STEAM OUT}, \text{ W/ W$_T$} \]
   3. destruction of exergy occur
      \[ \text{HXR}, \text{ IN TURBINE} \]
Cons. of energy

\[
\frac{CE_{HS}}{dt} = \dot{m}_s \Delta h + \sum m_i (h_i + \Delta h_i) - \sum m_i (h_i + \Delta h_i) \\
\dot{W}_{T, out} = m_i (h_i - h_s)
\]

\[
h_1 = h(\text{STEAM}, T_1, P_1) \\
h_2 = h(\text{STEAM}, P_2, x_2)
\]

\[
h_4 = \frac{m_\text{s} (h_1 - h_2)}{m_\text{s}} \\
h_5 = h(\text{AIR}, T_1) \\
\text{only}!!
\]

\[
h_4 = 2838 \text{ KJ/kg} \\
\dot{W}_T = 872 \text{ kW}
\]

Cons. of energy on HXR

\[
\dot{Q}_1 = \dot{Q}_2 + \sum m_i (h_i + \Delta h_i) - \sum m_i (h_i + \Delta h_i) \\
\dot{Q}_1 = m_\text{i}(h_1) + m_\text{s}(h_3) - m_\text{i}(h_2) - m_\text{s}(h_4)
\]

\[
h_3 = h(\text{STEAM}, T_2, P_3) = 163.6 \text{ KJ/kg}
\]

\[
h_4 = 2838 \text{ KJ/kg} \\
\dot{W}_T = 872 \text{ kW}
\]
\[ h_{s_4} = 2263 \text{ kJ/kg} \rightarrow \dot{W}_{T,2} = 1151 \text{ kW} \]

\[ \eta_T = \frac{872 \text{ kW}}{1151 \text{ kW}} = 0.758 \]

(c) (i) Exergy in \( W \) / Air Flow, Steam Flow

(ii) " Out " " " " " " " " " TURBINE Power

(iii) Exergy Destruction in

- Heat Exchanger (\( Q \) thru \( \Delta T \))
- Turbine (\( \eta_T < 1 \))

(d) \[ \dot{A} = m_a (a_{f,1} - a_{f,2}) - m_s (a_{f,5} - a_{f,3}) - \dot{W}_T \]
\[ a_{f(1)} - a_{f(2)} = \left( h_1 - h_2^o - T_o (\Delta_1 - \Delta_2) \right) - \left[ h_2 - h_2^o - T_o (\Delta_2 - \Delta_2^o) \right] \]

\[ = (h_1 - h_2) - T_o (\Delta_1 - \Delta_2) \]

\[ \Delta_1 - \Delta_2 = \Delta^o (T_1) - \Delta^o (T_2) - R \ln \left( \frac{P_2}{P_1} \right) \]

\[ \Delta^o \neq \Delta \text{ THIS IS A TABLE THING FOR IDEAL CASES} \]

OR USE EES:

\[ \Delta_1 = \Delta \text{(AIR, } T_1, P_1) = 6.171 \text{ KJ/kg-k} \]

\[ \Delta_2 = \Delta \text{(AIR, } T_2, P_2) = 5.997 \text{ KJ/kg-k} \]

.: NET INPUT

\[ = \text{NET AIR INPUT} = m_a (a_{f(1)} - a_{f(2)}) = \ldots = 1722 \text{ KJ} \]

\[ a_{f(1,5)} - a_{f(1,3)} = (h_5 - h_3) - T_o (\Delta_5 - \Delta_3) \]

\[ h_3 = h_5 = \] \[ \Delta_3 = \Delta \text{(STEAM, } T_3, P_3) \]

\[ = 0.5588 \text{ KJ/kg-k} \]

\[ \Delta_5 = \Delta \text{(STEAM, } P_5, x_5) \]

\[ = 7.739 \text{ KJ/kg-k} \]

.: NET OUTPUT IN STEAM

\[ = m_5 (a_{f(5)} - a_{f(3)}) = \ldots = 199 \text{ KJ} \]

FROM (2)

\[ A_{DES} = 1722 \text{ KJ} - 199 \text{ KJ} - 872 \text{ KJ} = 651 \text{ KJ} \]

TO KNOW HOW MUCH IS DESTROYED IN HXR & TURBINE, MUST LOOK AT THOSE AS SEPARATE SYSTEMS.
\[ \frac{dA^D}{dt} = \sum (1 - \frac{T_i}{T_o}) A_i - W_{\text{out}} + \sum m_i (a_f) \frac{\Delta q_i}{\text{out}} - \dot{A}_{\text{DES, HR}} \]

\[ \dot{A}_{\text{DES, HR}} = \dot{m}_a (a_{f1}) + \dot{m}_s (a_{f3}) - \dot{m}_a (a_{f2}) - \dot{m}_s (a_{f4}) \]

\[ \dot{A}_{\text{DES, HR}} = \dot{m}_a (a_{f1} - a_{f2}) + \dot{m}_s (a_{f3} - a_{f4}) = \ldots = 385 \text{ kW} \]

**TURBINE:**

\[ \frac{dA_{\text{ms}}}{dt} = \sum (1 - \frac{T_i}{T_o}) A_i - W_{\text{out}} + \sum m_i (a_f) \frac{\Delta q_i}{\text{out}} - \dot{A}_{\text{DES, TUR}} \]

\[ \dot{A}_{\text{DES, TUR}} = \dot{m}_s (a_{f1} - a_{f5}) - W_{\text{T, out}} \]

\[ = \dot{m}_s (a_{f1} - a_{f5} - T [\Delta q_4 - \Delta q_5]) - W_{\text{T, out}} \]

\[ = \ldots = 267 \text{ kW} \]

**FINALLY**

<table>
<thead>
<tr>
<th>NET RATE EXERGY IN (AIR FLOW)</th>
<th>1722 kW</th>
<th>(100%)</th>
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**DISPOSITION OF EXERGY**

- **RATE EXERGY OUT**
  - POWER OUT     872 kW (50.6%)
  - WATER STREAM  199 kW (11.6%)

- **RATE EXERGY DES.**
  - HR   385 kW (22.4%)
  - TURBINE 267 kW (15.5%)

\[ 1722 \text{ kW} \]

(100%)