A rigid beam is supported by two vertical rods. Rod A has a diameter of $d_A = 25$ mm and rod B has a diameter of $d_B = 10.2$ mm. Both rods are made of steel $(E = 210$ GPa). For the $60$ kN force applied as shown,

a) find the reactions at $A$ and $B$, and
b) the displacements of each rod.

From (1)

$$F_A = P - F_B = (60 - 20) \text{kN} = 40 \text{kN}$$

b) FBD Rod A with cut:

$$\sigma = \frac{F_A}{A_A} \quad \text{Nucke's Law:}$$

$$\sigma_A = \frac{F_A}{A_A} \quad (3) \equiv (4)$$

$$\sigma_A = \frac{F_A}{A_A}$$

$$\delta = \frac{F_A L_1}{(E) \pi d_A^2} = \frac{(40 \times 10^3 \text{N})(3 \text{m})}{(210 \times 10^9 \text{N} \cdot \text{m}^{-2})(0.025 \text{m})^2} = 1.164 \text{mm}$$

Similarly...

$$\delta_B = \frac{F_B L_2}{E \pi d_B^2} = \frac{(20 \times 10^3 \text{N})(2 \text{m})}{(210 \times 10^9 \text{N} \cdot \text{m}^{-2})(0.0102 \text{m})^2} = 2.331 \text{mm}$$
NOTE HOW IT DEFLECTS

\[ \delta_b = 2 \delta_a \]
Example

Two steel \((E=30 \times 10^3\text{ ksi})\) rods both with cross sectional area \(A=1.0\text{ in}^2\) are used to support a rigid beam connected to a wall via a smooth pin. A 10 kip point load is applied to the beam at the location shown. Neglecting the weight of the beam, find the tension in each rod.

Two EQNS, 3 UNKNOWNS!
FBDs of RODS DON'T HELP \(\Rightarrow\) STATICAELY INDETERMINATE.

MUST LOOK @ GEOMETRY & DEFORMATION.

BECAUSE BEAM IS RIGID:

\[
\frac{\delta_B}{\delta_C} = \frac{2a+b}{b} \quad (3)
\]

ADDED TWO UNKNOWNS! LOOK AT RODS A & C

\[
\sigma_A = \frac{F_A L_A}{E_A A_A} \quad (4)
\]

\[
\sigma_C = \frac{F_C L_C}{E_C A_C} \quad (5)
\]

TWO MORE EQNS, NO NEW UNKNOWNS!
(4) & (5) INTO (3)

\[ \frac{F_{AI_1}}{EA} \left( \frac{1}{2a+b} \right) = \frac{F_{EI_2}}{EA} \left( \frac{1}{b} \right) \quad F_c = \frac{b}{2a+b} \left( \frac{L_1}{L_2} \right) F_a \quad (6) \]

SUB INTO (2)

\[ -(2a+b)F_a + (a+b)p - b \frac{b}{2a+b} \left( \frac{L_1}{L_2} \right) F_a = 0 \]

\[ F_a = \frac{(a+b)}{(2a+b) + \frac{b^2}{(2a+b)} \left( \frac{L_1}{L_2} \right)} \]

\[ = \frac{20'' + 60''}{(40'' + 60'')} + \frac{(60'')^2}{(40'' + 60'')} \left( \frac{140''}{60''} \right) \]

\[ = \frac{6.451 \text{ kips}}{10 \text{ kips}} \]

FROM (6)

\[ F_c = \frac{60''}{40'' + 60''} \left( \frac{140''}{60''} \right) \left( 6.451 \text{ kips} \right) = 2.581 \text{ kips} \]

NEW!

FIND STRESS IN EA. ROD E \( \Theta \).

\[ \sigma_a = \frac{F_{AI_1}}{A} = \frac{6.45 \text{ kips}}{1 \text{ in}^2} = 6.45 \text{ ksi} \]

\[ \sigma_b = \frac{F_{EI_2}}{A} = \ldots = 2.58 \text{ ksi} \]

\[ \Theta \approx \tan^{-1} \left( \frac{\sigma_a}{2a+b} \right) = \tan^{-1} \left( \frac{F_{AI_1}}{EA (2a+b)} \right) \]

\[ = \tan^{-1} \left( \frac{6.451 \text{ kips} \times 40''}{30 \times 0.25 \text{ kips in}^{-2} \times (140'' + 60'')} \right) = 0.00493^\circ \]
\[ \tan^{-1} \left( \frac{\Delta \theta}{a} \right) = \theta = \tan^{-1} \left( \frac{0.0129 m}{2 m} \right) = 0.36^\circ \] (a)

**NOTE:**
\[ \varepsilon = \frac{\Delta \alpha}{L} = \frac{0.0129 m}{5 m} = 0.0025 = 0.2\% ! \]

**BIG STRAIN! REMEMBER YIELD?** DEFINED \( \varepsilon \to 0.02\% \).

\[ \sigma_a = \frac{F_a}{A_a} = \frac{36 \text{ KN}}{(200) \times 10^{-5} \text{ m}^2} = 180 \text{ MPa} \]

\[ \sigma_b = \frac{F_b}{A_b} = \ldots = 180 \text{ MPa} \]

CHECK IF IT'S CLOSE TO \( \sigma_{\text{YIELD}} \)
Example

A rigid, weightless beam is supported by a smooth pin at B. Two aluminum \((E=70 \text{ GPa})\) rods, both with cross sectional area \(A=200 \text{ mm}^2\), also support the rod at pins A and C. For the 24 kN load at D,

a) find the rotation angle of the rod,

b) the force in each rod, and

c) the stress in each rod.

\[\Sigma F_x = 0 \quad B_x = 0\]
\[\Sigma F_y = 0 \quad -F_A + B_y + F_c - P = 0 \quad (1)\]
\[\Sigma M_B = 0\]
\[aF_A + aF_c - (a+b)P = 0 \quad (2)\]
\[a(F_A + F_c) - (a+b)P = 0\]

**GEOMETRY & DEFORMATION**

\[\delta_A = \delta_C \quad (3) \quad \theta = \pm \arctan \left( \frac{d_A}{a} \right)\]

**STRESS/STRAIN**

\[\sigma_A = \frac{F_A L_1}{E_A A_A} \quad (4)\]
\[\sigma_F = \frac{F_c L_2}{E_B A_B} \quad (5)\]

(4), (4) & (5) GIVE

\[\frac{F_A L_1}{E_A A_A} = \frac{F_c L_2}{E_B A_B}\]
\[F_A = F_c\]

**SUB INTO (2)**

\[2aF_A = (a+h)P = 0\]
\[F_A = \frac{(a+b)P}{2a} = \frac{(6 \text{ m})(24 \text{ kN})}{(4 \text{ m})} = 36 \text{ kN}\]

**F = F_A**

\[F_c = F_A = 36 \text{ kN}\]

**FROM (4)**

\[\delta_A = \frac{(36 \text{ kN})(5 \text{ m})}{(70 \text{ GPa})(200) \times 10^{-6} \text{ m}^2} = 0.0129 \text{ m}\]
\[= 12.9 \text{ mm}\]