Periods of Curves with Automorphisms

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- *Information Theory suggests that the period message is compressible*

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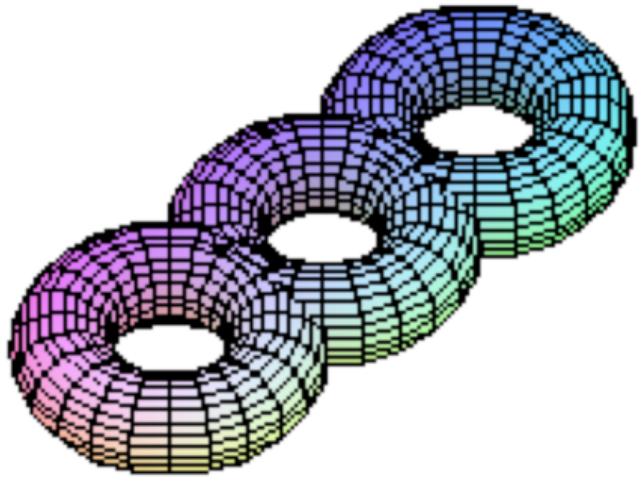
- Not like this . . .



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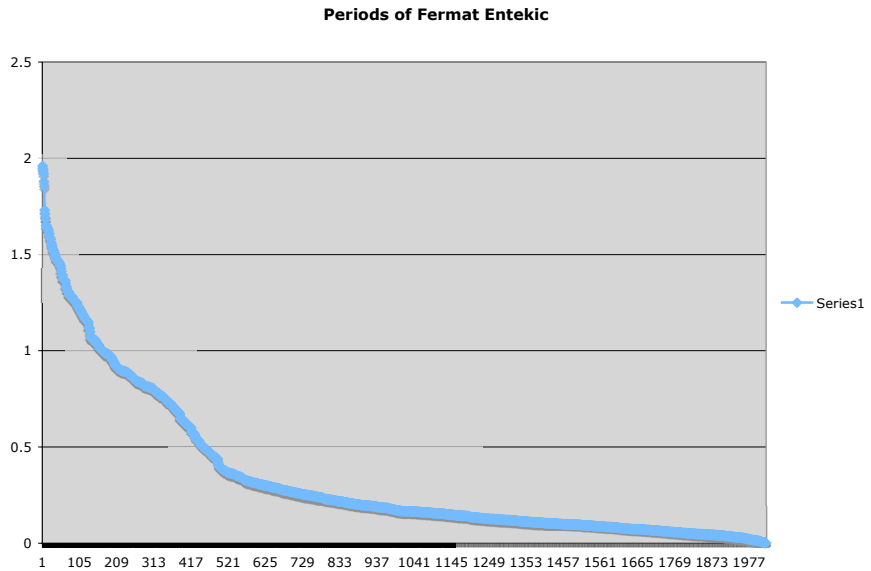
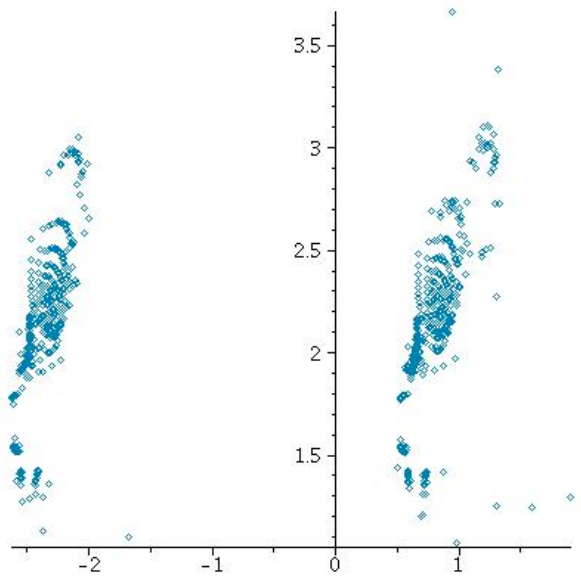
- Virtually *any* distribution corresponds to an abelian variety
- But (experimentally) Jacobians look different:



Hyperelliptics

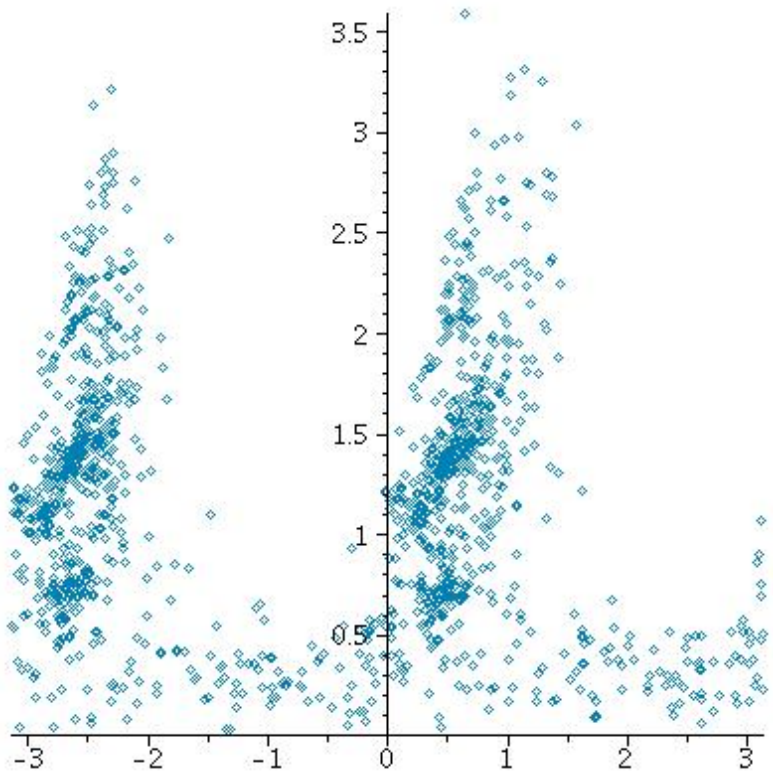
Hyperelliptic periods are *band–limited*



Arguments of “random" hyperelliptic, genus 39

Trigonals

This is not the case with a “random" trigonal

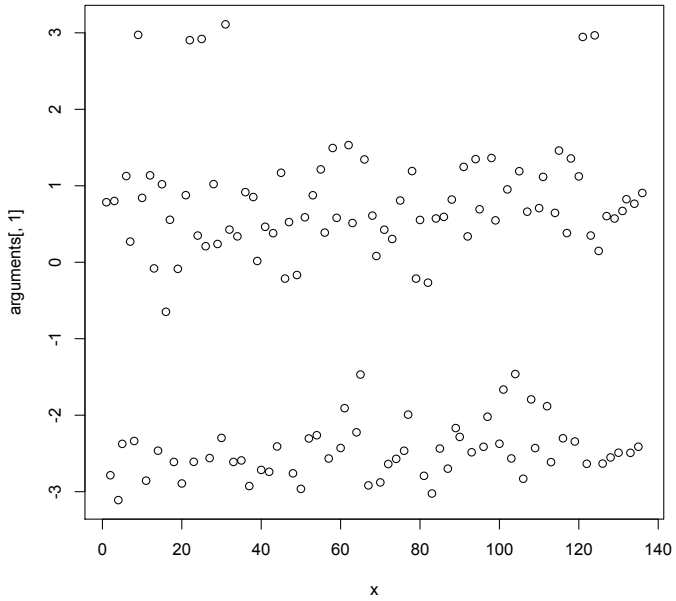


Arguments of a “random" trigonal curve

Exceptions?

Nakajima, Ryo, “On splitting certain Jacobian varieties," *J. Math. Kyoto Univ.* **42–7** (2007), 391–415.

$$y^3 = \frac{1 + x^m}{1 - tx^m},$$



Period Arguments, $m = 9$

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THANK YOU!