

# Structure of $\widetilde{\mathcal{M}}_4$

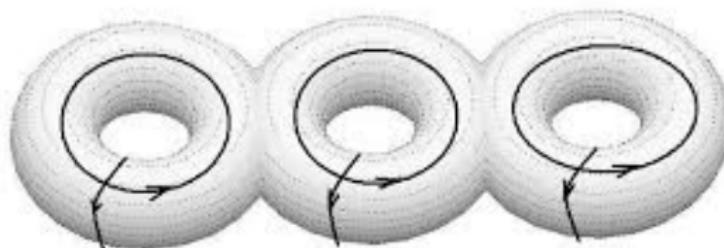
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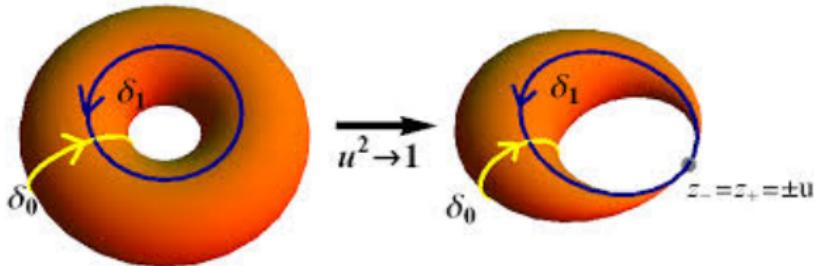
April 15th, 2018

## Homology basis

How many "differents" loops you may draw in the surface?

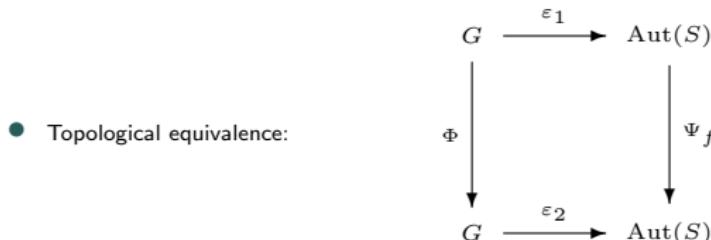


Play with the curves



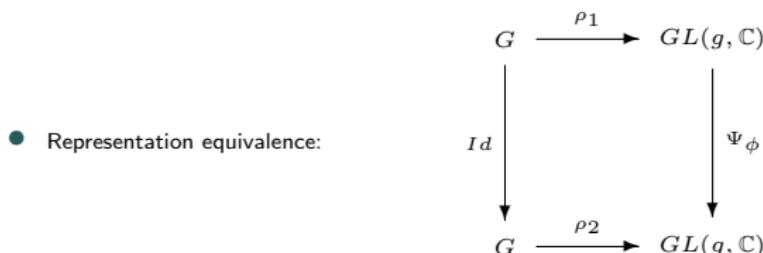
# Equivalence actions

Let  $G$  be a finite group,  $S$  be a compact Riemann surface



$\Phi \in \text{Aut}(G)$ ,  $\Psi_f$  conjugation by an orientation preserving homeomorphism of  $S$ .

- Conformal equivalence  $\Psi_f$  conjugation by an isomorphism of  $S$ .



$\Psi_\phi$  conjugation by lineal isomorphism of  $\mathbb{C}^g$ .

- Kimura-Kuribayashi equivalence [Conflated – A.Broughton]  $\rho_1(G)$  and  $\rho_2(G)$  are conjugated subgroup of  $GL(g, \mathbb{C})$ .

## Some references about genus 4

- I.Kuribayashi and A.Kuribayashi, On Automorphism Group of Compact Riemann Surfaces of Genus 4. Proc. Japan Acad., 62, Ser. A, pag. 65-68 (1986)
- H.Kimura, Classification of automorphism groups, up to topological equivalence, of compact Riemann surfaces of genus 4. *J. Algebra* 264 (2003), no. 1, 26-54.
- A.Costa and M.Izquierdo Equisymmetric strata of singular locus of the moduli space of Riemann surfaces of genus 4, Geometry of Riemann surfaces, London Math. Society **368**, pag. 120-138 (2010)
- Bartolini G., Costa A. F. and Izquierdo, M. On the orbifold structure of the moduli space of Riemann surfaces of genera four and five. Rev. R. Acad. Cienc. Exactas Fis. Nat. Ser. A Math. RACSAM 108, No. 2, 769793, (2014).

# Families in $\mathcal{M}_4$

① The 7-dimensional family:

$$(\Gamma(0; [2, 2, 2, 2, 2, 2, 2]), C_2, \Phi_2)$$

② The 6-dimensional family:

$$(\Gamma(1; [2, 2, 2, 2, 2, 2]), C_2, \Phi_3)$$

③ The 5-dimensional family:

$$(\Gamma(2; [2, 2]), C_2, \Phi_4)$$

④ The 3-dimensional families:

$$(\Gamma(0; [3, 3, 3, 3, 3, 3]), C_3, \Phi_7), \quad (\Gamma(0; [3, 3, 3, 3, 3, 3]), C_3, \Phi_8)$$

$$(\Gamma(0; [2, 2, 2, 2, 2, 2]), C_4, \Phi_9), \quad (\Gamma(0; [2, 2, 2, 2, 2, 2]), D_3, \Phi_{51})$$

$$(\Gamma(1; [3, 3, 3]), C_3, \Phi_6), \quad (\Gamma(1; [2, 2, 2]), C_2 \times C_2, \Phi_{33}), \quad (\Gamma(2; --, C_3, \Phi_5)$$

# 2–dimensional families

- 1**  $s = (0; [2, 4, 4, 4, 4]), \quad G \cong C_4, \text{ i) } \Phi_{10}.$
- 2**  $s = (0; [2, 2, 2, 3, 6]), \quad G \cong C_6, \text{ i) } \Phi_{17}.$
- 3**  $s = (0; [2, 2, 3, 3, 3]), \quad G \cong C_6, \text{ i) } \Phi_{18}.$
- 4**  $s = (0; [2, 2, 2, 2, 4]), \quad G \cong D_4, \text{ i) } \Phi_{34}, \text{ ii) } \Phi_{35}.$
- 5**  $s = (0; [2, 2, 3, 3, 3]), \quad G \cong D_3, \text{ i) } \Phi_{53}.$
- 6**  $s = (0; [2, 2, 2, 2, 2]), \quad G \cong D_6, \text{ i) } \Phi_{56}.$
- 7**  $s = (1; [4, 4]), \quad G \cong C_4, \text{ i) } \Phi_{11}.$
- 8**  $s = (1; [2, 2]), \quad G \cong C_6, \text{ i) } \Phi_{16}.$
- 9**  $s = (1; [2, 2]), \quad G \cong D_3, \text{ i) } \Phi_{52}.$

# Maximal 2–dimensional families

- 1**  $s = (0; [2, 4, 4, 4, 4]), \quad G \cong C_4, \text{ i) } \Phi_{10}.$
- 2**  $s = (0; [2, 2, 2, 3, 6]), \quad G \cong C_6, \text{ i) } \Phi_{17}.$
- 3**  $s = (0; [2, 2, 3, 3, 3]), \quad G \cong C_6, \text{ i) } \Phi_{18}.$
- 4**  $s = (0; [2, 2, 2, 2, 4]), \quad G \cong D_4, \text{ i) } \Phi_{34}, \text{ ii) } \Phi_{35}.$
- 5**  $s = (0; [2, 2, 3, 3, 3]), \quad G \cong D_3, \text{ i) } \Phi_{53}.$
- 6**  $s = (0; [2, 2, 2, 2, 2]), \quad G \cong D_6, \text{ i) } \Phi_{56}.$

# 1-dimensional families

- 1  $s = (0; [5, 5, 5, 5]), \quad G \cong C_5, \text{ i) } \Phi_{12}, \text{ ii) } \Phi_{13}, \text{ iii) } \Phi_{14}.$
- 2  $s = (0; [2, 6, 6, 6]), \quad G \cong C_6, \text{ i) } \Phi_{15}.$
- 3  $s = (0; [3, 3, 6, 6]), \quad G \cong C_6, \text{ i) } \Phi_{19}, \text{ ii) } \Phi_{20}.$
- 4  $s = (0; [2, 2, 8, 8]), \quad G \cong C_8, \text{ i) } \Phi_{21}.$
- 5  $s = (0; [2, 4, 4, 4]), \quad G \cong Q_8, \text{ i) } \Phi_{36}.$
- 6  $s = (0; [3, 3, 3, 3]), \quad G \cong C_3 \times C_3, \text{ i) } \Phi_{40}, \text{ ii) } \Phi_{41}.$
- 7  $s = (0; [2, 2, 5, 5]), \quad G \cong D_5, \text{ i) } \Phi_{42}, \text{ ii) } \Phi_{43}.$
- 8  $s = (0; [2, 2, 5, 5]), \quad G \cong C_{10}, \text{ i) } \Phi_{23}.$
- 9  $s = (0; [2, 3, 3, 3]), \quad G \cong \mathfrak{A}_4, \text{ i) } \Phi_{59}.$
- 10  $s = (0; [2, 2, 3, 6]), \quad G \cong D_6, \text{ i) } \Phi_{57}.$
- 11  $s = (0; [2, 2, 3, 6]), \quad G \cong C_6 \times C_2, \text{ } \Phi_{54}.$
- 12  $s = (0; [2, 2, 2, 8]), \quad G \cong D_8, \text{ i) } \Phi_{37}.$
- 13  $s = (0; [2, 2, 3, 3]), \quad G \cong C_3 \times \mathfrak{S}_3, \text{i) } \Phi_{60}, \text{ ii) } \Phi_{61}.$
- 14  $s = (0; [2, 2, 3, 3]), \quad G \cong C_3 \times C_3 * C_2, \text{ i) } \Phi_{64}.$
- 15  $s = (0; [2, 2, 2, 5]), \quad G \cong D_{10}, \text{ i) } \Phi_{44}.$
- 16  $s = (0; [2, 2, 2, 4]), \quad G \cong \mathfrak{S}_4, \text{ i) } \Phi_{66}.$
- 17  $s = (0; [2, 2, 2, 3]), \quad G \cong \mathfrak{S}_3 \times \mathfrak{S}_3, \text{i) } \Phi_{71}.$
- 18  $s(1; [2]), \quad G \cong \mathfrak{A}_4, \text{ i) } \Phi_{58}.$

# Maximal 1–dimensional families

- 1  $s = (0; [2, 2, 5, 5]), \quad G \cong D_5, \Phi_{43}.$
- 2  $s = (0; [2, 2, 2, 5]), \quad G \cong D_5, \Phi_{44}.$
- 3  $s = (0; [5, 5, 5, 5]), \quad G \cong C_5, \Phi_{12}.$
- 4  $s = (0; [2, 2, 3, 6]), \quad G \cong C_6 \times C_2, \Phi_{54}.$
- 5  $s = (0; [2, 2, 3, 6]), \quad G \cong D_6, \Phi_{57}.$
- 6  $s = (0; [2, 6, 6, 6]), \quad G \cong C_6, \Phi_{15}.$
- 7  $s = (0; [2, 2, 2, 8]), \quad G \cong D_8, \Phi_{37}.$
- 8  $s = (0; [2, 4, 4, 4]), \quad G \cong Q_8, \Phi_{36}.$
- 9  $s = (0; [2, 2, 2, 3]), \quad G \cong \mathfrak{S}_3 \times \mathfrak{S}_3, \Phi_{71}.$
- 10  $s = (0; [2, 3, 3, 3]), \quad G \cong A_4, \Phi_{59}.$
- 11  $s = (0; [2, 2, 2, 4]), \quad G \cong \mathfrak{S}_4, \Phi_{66}.$
- 12  $s = (0; [2, 2, 3, 3]), \quad G \cong C_3 \times \mathfrak{S}_3, \text{i) } \Phi_{60}, \text{ ii) } \Phi_{61}.$

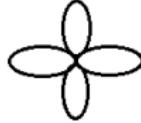
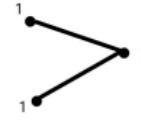
# Limit points of families with dimension 1

case	multicurves	Orbifolds	Vertex	Edges	Degree	Weight	Graph
1	$\gamma_{1,2}^*$	$\mathcal{O}_{3,4}^*$	2	5	5	0	
2	$\gamma_{1,2}^* (\gamma_{1,3}^*)$	$\mathcal{O}_{3,4}^*$	2	5	5	0	
2	$\gamma_{2,3}^*$	$\mathcal{O}_{1,4}^*$	2	1	1	2	
3	$\gamma_{1,2}$	$\mathcal{O}_{1,2}, \mathcal{O}_{3,4}$	$1+1=2$	1	1,1	2,2	

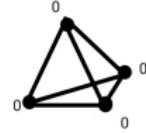
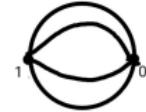
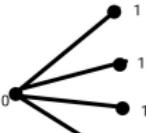
# Limit points of families with dimension 1

case	multicurves	Orbifolds	Vertex	Edges	Degree	Weight	Graph
4	$\gamma_{1,2}^*$	$\mathcal{O}_{3,4}^*$	2	3	3	1	
4	$\gamma_{2,3}$	$\mathcal{O}_{2,3}, \mathcal{O}_{1,4}$	2+1	2	1, 2	1, 2	
5	$\gamma_{1,2}^*$	$\mathcal{O}_{3,4}^*$	2	1	1	2	
5	$\gamma_{1,3} (\gamma_{2,3})$	$\mathcal{O}_{1,3}, \mathcal{O}_{2,4}$	2+1=3	6	3,6	0,0	

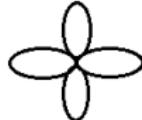
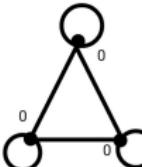
# Limit points of families with dimension 1

case	multicurves	Orbifolds	Vertex	Edges	Degree	Weight	Graph
6	$\gamma_{1,2}$	$\mathcal{O}_{1,2}, \mathcal{O}_{3,4}$	1+1	2	2,2	1,2	
7	$\gamma_{1,2}^*$	$\mathcal{O}_{3,4}^*$	1	4	8	0	
7	$\gamma_{1,4}^*$	$\mathcal{O}_{2,3}^*$	2	1	1	2	
8	$\gamma_{1,2}$	$\mathcal{O}_{1,2}, \mathcal{O}_{3,4}$	$2+1=3$	2	1,2	1,2	

# Limit points of families with dimension 1

case	multicurves	Orbifolds	Vertex	Edges	Degree	Weight	Graph
9	$\gamma_{1,2}^*$	$\mathcal{O}_{3,4}^*$	2	3	3	1	
9	$\gamma_{1,4}^*$	$\mathcal{O}_{2,3}^*$	6	9	3	0	
10	$\gamma_{1,2}$	$\mathcal{O}_{1,2}, \mathcal{O}_{3,4}$	1+1	4	4,4	0,1	
10	$\gamma_{1,4}$	$\mathcal{O}_{1,4}, \mathcal{O}_{2,3}$	1+4=5	4	4,1	0,1	

# Limit points of families with dimension 1

case	multicurves	Orbifolds	Vertex	Edges	Degree	Weight	Graph
11	$\gamma_{2,3}^*$	$\mathcal{O}_{1,4}^*$	1	4	8	0	
11	$\gamma_{1,2}^*$	$\mathcal{O}_{3,4}^*$	3	6	4	0	
12	$\gamma_{1,2}^*$	$\mathcal{O}_{3,4}^*$	2	3	3	1	

## Limit points of families with dimension 1

case	multicurves	Orbifolds	Vertex	Edges	Degree	Weight	Graph
12	$\gamma_{1,4}$	$\mathcal{O}_{1,4}, \mathcal{O}_{2,3}$	$1+3=4$	3	3,1	1,1	
12	$\gamma_{1,2}^*$	$\mathcal{O}_{3,4}^*$	6	9	3	0	
12	$\gamma_{1,4}$	$\mathcal{O}_{1,4}, \mathcal{O}_{2,3}$	$1+1=2$	3	1,1	1,1	

# References

-  A. Broughton The equisymmetric stratification of the moduli space and the Krull dimension of mapping class groups.  
Topology and its application. 37, 101-113, 1990.
-  R.Díaz and V.González-Aguilera Limit points of the branch locus of  $\mathcal{M}_g$ . arXiv 1703.07328v1
-  D. Singerman Finitely Maximal Fuchsian groups. J. London Math. Soc. 6 , no. 2, 29-28. (1972)

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