

Macbeath Singerman Symmetries of Quasi-platonic $PSL_2(q)$ -actions

Remark 1 $\mathcal{M}_{B_p} = \mathcal{M}(B_p, p)$ is the set of ovals of \mathcal{M}_p whose edge pattern contains edges of type p . Similarly for $\mathcal{M}_{B_q}, \mathcal{M}_{B_r}$.

Symmetries of $PSL_2(7)$ actions

signature (l, m, n)	genus	#actions	inner/outer $\theta \in PSL_2(q)?$	“symmetry types” $(\pm \mathcal{M}_{B_p} , \pm \mathcal{M}_{B_q} , \pm \mathcal{M}_{B_r})$
(2, 3, 7)	3	1	outer	(-1, -1, -1)
(2, 4, 7)	10	1	outer	(-1, -1, -1)
(2, 7, 7)	19	1	outer	(-1, -1, -1)
(3, 3, 4)	8	1	inner	(-1, -1, -1)
		1	outer	(-1, -1, -1)
(3, 3, 7)	17	1	inner	(-1, -1, -1)
(3, 4, 4)	15	1	inner	(-1, -1, -1)
(3, 4, 7)	24	1	outer	(-3, -3, -3)
		1	outer	(-1, -1, -1)
(3, 7, 7)	33	1	outer	(-2, -2, -2)
		1	outer	(-3, -3, -3)
(4, 4, 4)	22	1	outer	(-1, -1, -1)
		1	inner	(-1, -1, -1)
(4, 4, 7)	31	1	outer	(-3, -1, -3)
(4, 7, 7)	40	2	outer	(-1, -1, -1)
(7, 7, 7)	49	1	outer	(-3, -3, -3)

Symmetries of $PSL_2(8)$ actions

signature (l, m, n)	genus	#actions	inner/outer $\theta \in PSL_2(q)?$	“symmetry types” $(\pm \mathcal{M}_{B_p} , \pm \mathcal{M}_{B_q} , \pm \mathcal{M}_{B_r})$
(2, 3, 7)	7	1	inner	(-2, -2, -2)
(2, 3, 9)	15	1	inner	(-2, -2, -2)
(2, 7, 7)	55	2	inner	(-2, -2, -2)
(2, 7, 9)	63	3	inner	(-2, -2, -2)
(2, 9, 9)	71	1	inner	(-2, -2, -2)
(3, 3, 7)	41	1	inner	(-4, -4, -4)
(3, 3, 9)	57	1	inner	(-4, -4, -4)
(3, 7, 7)	97	3	inner	(-4, -4, -4)
(3, 7, 9)	105	3	inner	(-4, -4, -4)
(3, 9, 9)	113	1	inner	(-4, -4, -4)
(7, 7, 7)	145	4	inner	(-4, -4, -4)
(7, 7, 9)	153	9	inner	(-4, -4, -4)
(7, 9, 9)	161	9	inner	(-4, -4, -4)
(9, 9, 9)	169	6	inner	(-4, -4, -4)

Samples from $PSL_2(11)$, $PSL_2(13)$, $PSL_2(19)$, $PSL_2(23)$

q	$ PSL_2(q) $	signature (l, m, n)	genus	#actions	inner/outer $\theta \in PSL_2(q)$?	“symmetry types”
11	660	(5, 5, 5)	34	1 3 4	outer outer inner	(-2, -2, -2) (-5, -5, -5) (-6, -6, -6)
11	660	(11, 11, 11)	241	1	outer	(-5, -5, -5)
13	1092	(13, 13, 13)	241	1	inner	(-6, -6, -6)
19	3420	(2, 5, 10)	343	1 1	inner outer	(-1, -1, -1) (-1, -3, -3)
19	3420	(10, 10, 10)	1198	10 1 1 1 1 1 1	inner inner inner inner inner inner inner	(-1, -1, -1) (-1, -2, -2) (-2, -1, -2) (-2, -2, -1) (-5, -2, -2) (-2, -5, -2) (-2, -2, -5)
19	3420	(19, 19, 19)	1441	1	outer	(-9, -9, -9)
23	6072	(2, 4, 12)	507	1 1	outer inner	(-1, -1, -1) (-2, -1, -3)
23	6072	(11, 11, 11)	2209	46 3 11 8 54 77	outer inner inner inner outer inner	(-2, -2, -2) (-4, -4, -4) (-6, -6, -6) (-8, -8, -8) (-11, -11, -11) (-12, -12, -12)
23	6072	(23, 23, 23)	2641	1	outer	(-11, -11, -11)