Quantum Computing & Pell's Equation

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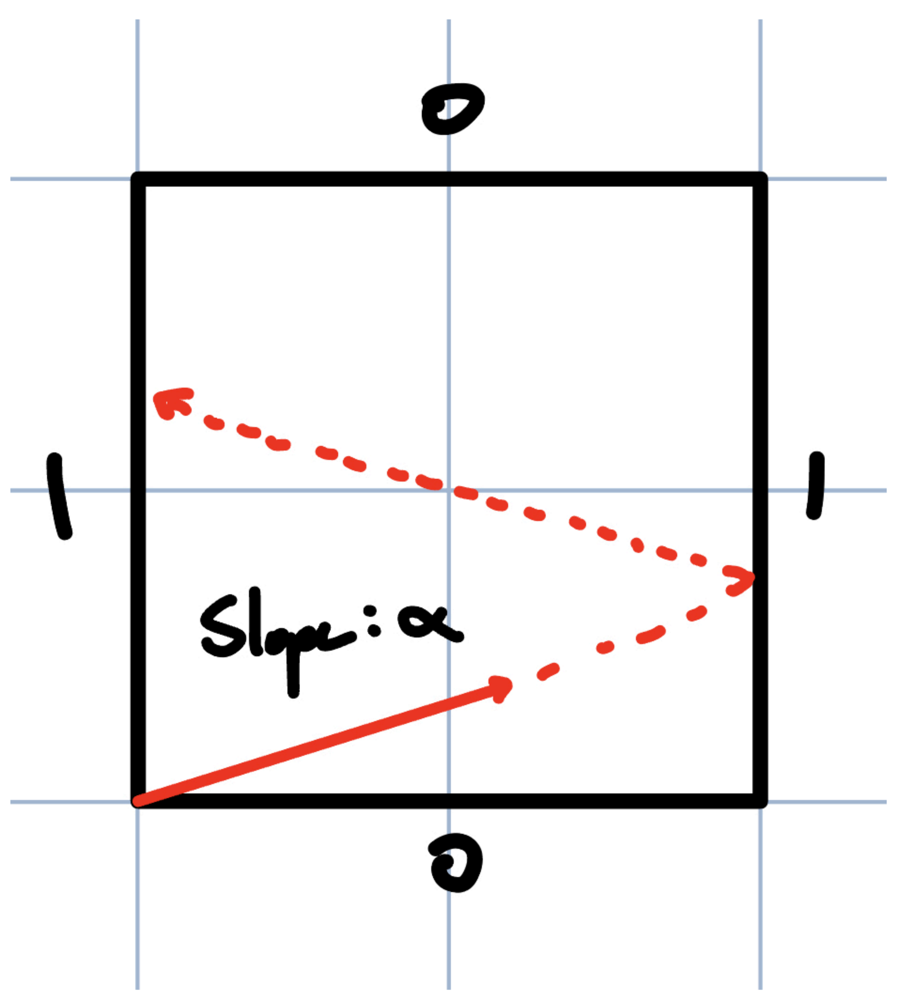
Problem

Given the cutting sequence of the billiard ball's movement, can **Hallgren's algorithm** be used to **calculate the pseudo-period** of $f(x)$ in order to **find the irrational slope** value in which the billiard ball was put in motion?

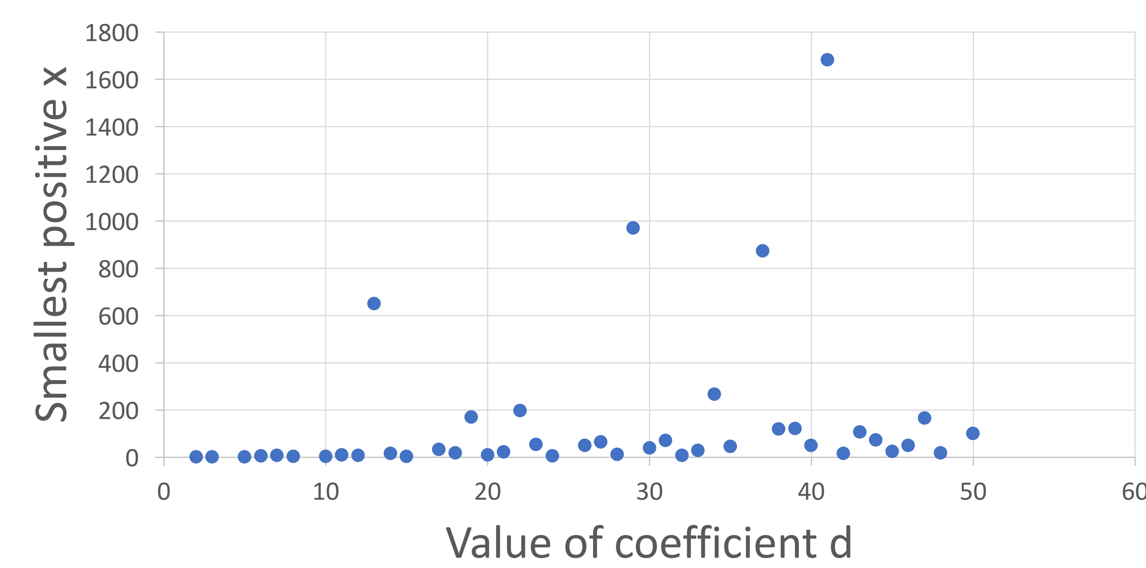
Billiard table: unit square



- Every time the ball hits a horizontal wall, record 0.
- Every time it hits a vertical wall, record 1.
- Slope is irrational! We want to find this slope value.

Pell's equation

$$x^2 - dy^2 = 1$$



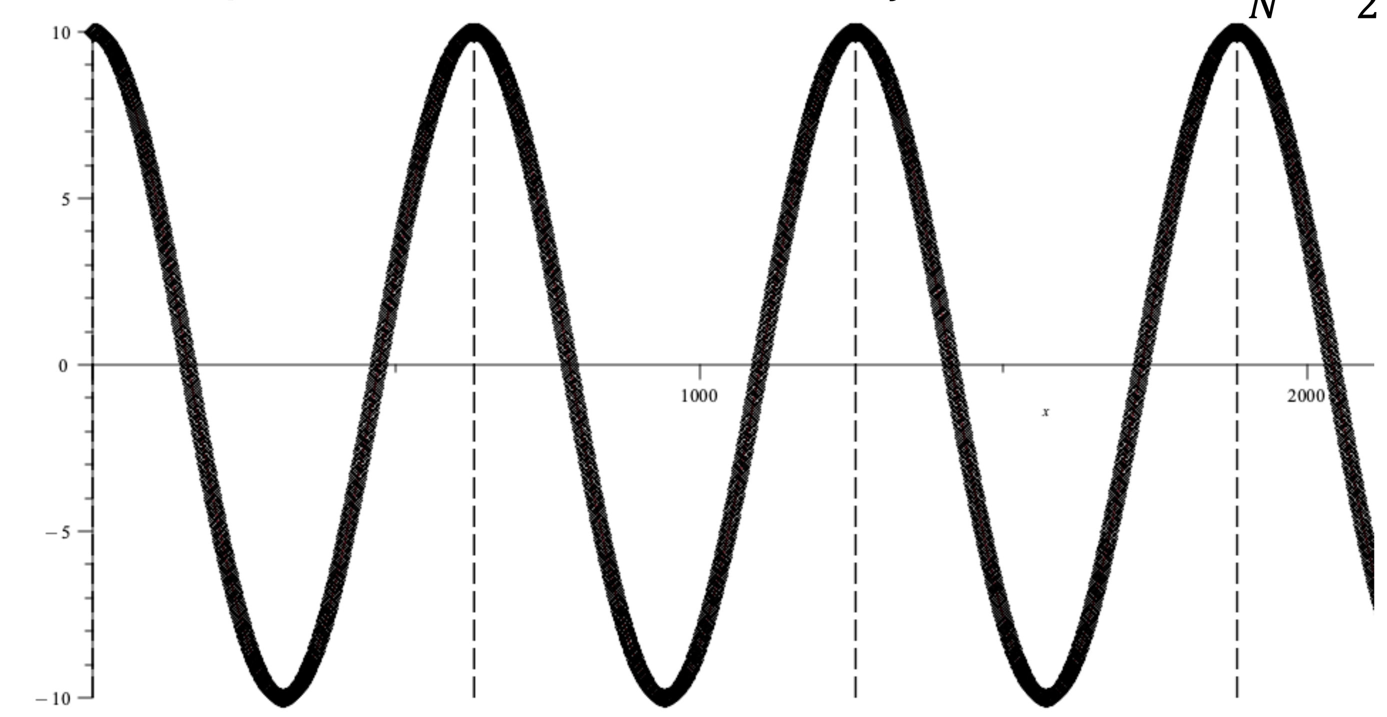
Smallest positive x value that satisfies Pell's equation for coefficient d

- Pell's equation has no obvious patterns between coefficient d and least positive solutions (x, y).
- Hallgren's quantum algorithm can be used to efficiently compute the solution to Pell's equation.

Pseudo-periodicity

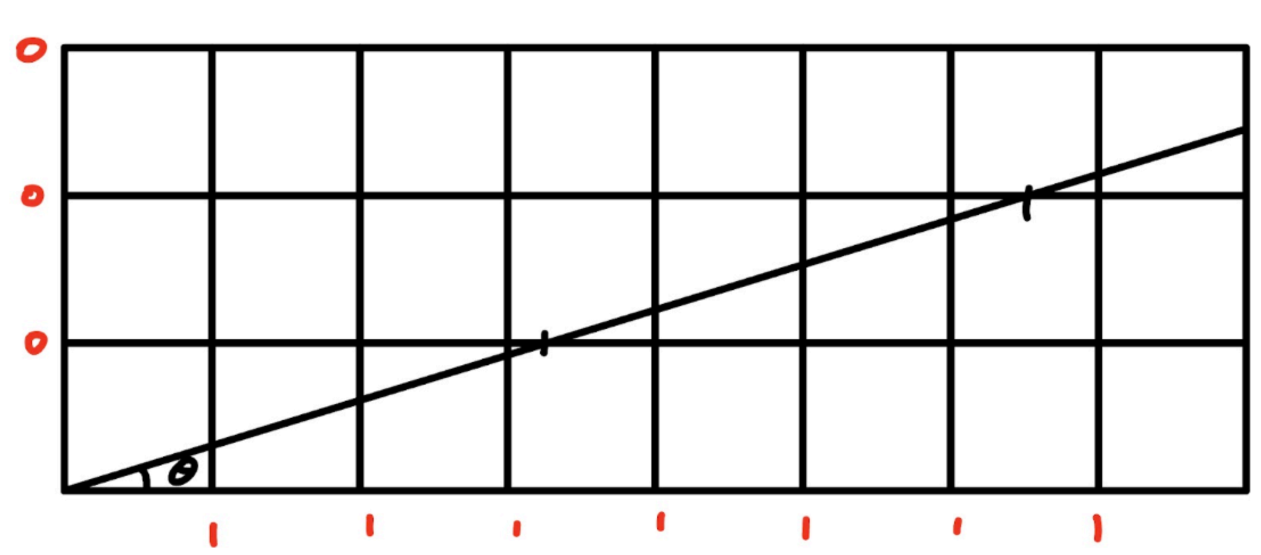
Definition: Either $f(k) = f(k + \lfloor iS \rfloor)$ or $f(k) = f(k + \lceil iS \rceil)$ for all integer values of i and $S \in \mathbb{R}$.

Example: Let $S = 200\pi, k = 0, f(x) = 10\sin\frac{x}{N} + \frac{\pi}{2}$.



This function satisfies the definition of pseudo-periodicity.
We must find a corresponding function that generates the cutting sequence of a billiard ball.

Cutting sequences (Expanded billiard table)

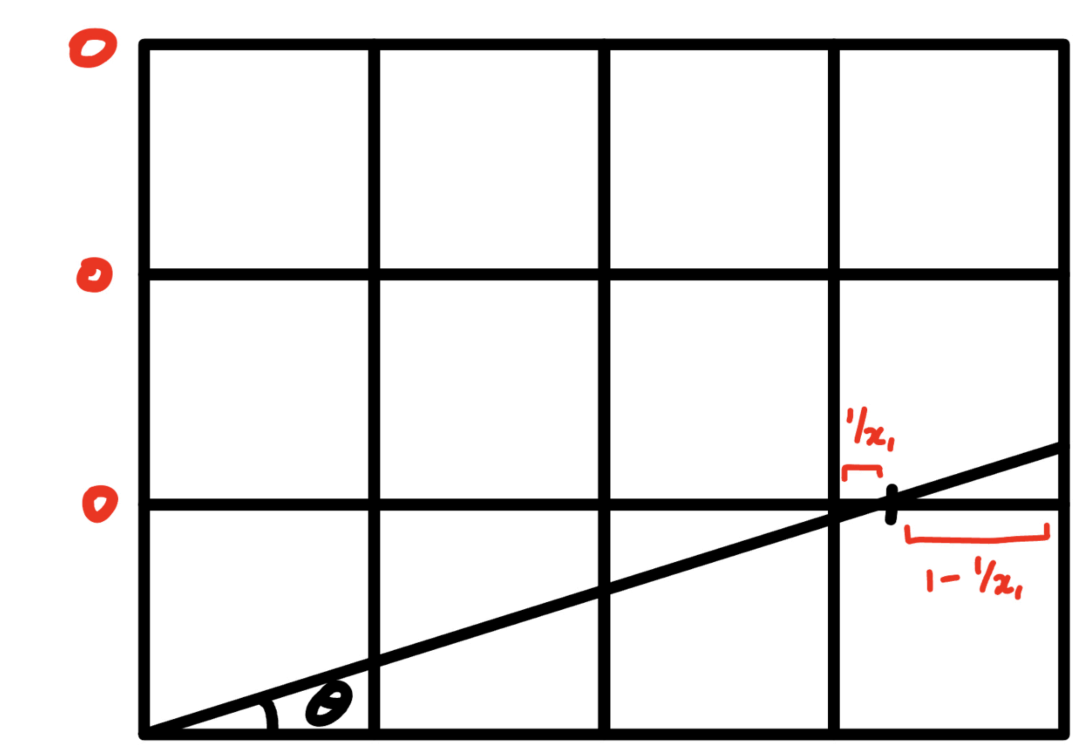


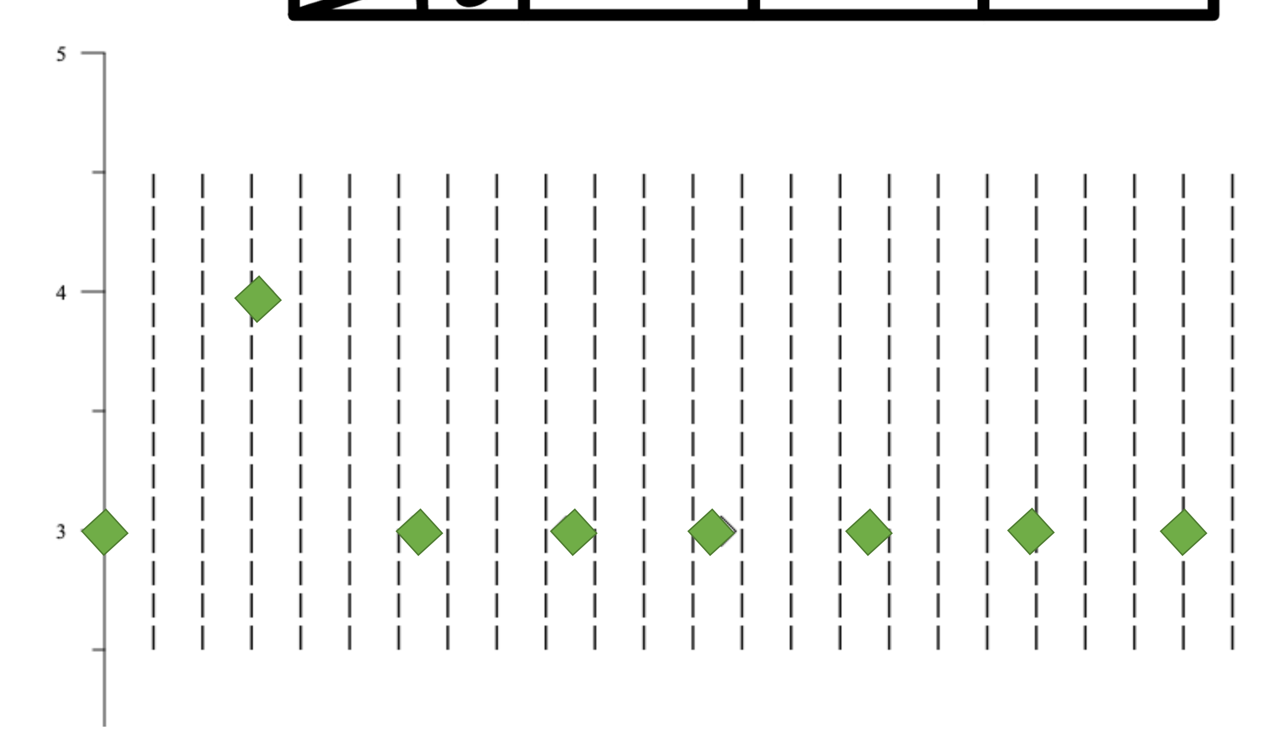
The cutting sequence of this example is $1110\,1110$.

Our $f(x)$

$$\frac{1}{\theta} = \left\lfloor \frac{1}{\theta} \right\rfloor + \frac{1}{x_1}$$

$$f(n) = \left\lfloor \frac{n}{m} \right\rfloor - \left\lfloor \frac{n-1}{m} \right\rfloor$$





The dashed lines indicate the pseudo-period of the function.

Points are far too close to pseudo-period lines.

Still a work in progress!

Parallel comparisons

Billiard ball problem

1. Cutting sequence produces some $f(x)$ (Our contribution).
2. Input into Hallgren's algorithm.
3. Produce pseudo-period to find the irrational slope value for billiard ball launch.

Pell's equation

1. Pell's equation produces $f(x)$.
2. Input into Hallgren's algorithm.
3. Produce pseudo-period to find least positive solution (x_1, y_1) for solving Pell's.