

What is the recurrence relation for the coin-row problem?

The coin-row problem is a dynamic programming problem that involves choosing a subset of coins from a given row such that the sum of the chosen coins is maximized while adhering to a constraint that no two adjacent coins can be chosen. The recurrence relation for this problem can be defined as:

$$F(i) = \max \{F(i-1), F(i-2) + v(i)\}$$

Where  $F(i)$  is the maximum value that can be obtained from the first  $i$  coins,  $v(i)$  is the value of the  $i$ -th coin, and  $i$  is the index of the coin. The recurrence relation is defined for  $i = 1$  to  $n$ , where  $n$  is the total number of coins.

The base case for this recurrence relation is  $F(0) = 0$ , which represents that no coins can be chosen if there are no coins in the row.

In this recurrence relation,  $F(i)$  is the maximum value that can be obtained from the first  $i$  coins. The maximum value can be obtained by either choosing the  $i$ -th coin or not choosing the  $i$ -th coin. If the  $i$ -th coin is chosen, then the maximum value is the value of the coin plus the maximum value that can be obtained from the first  $i-2$  coins (to maintain the constraint that no two adjacent coins can be chosen). If the  $i$ -th coin is not chosen, then the maximum value is the maximum value that can be obtained from the first  $i-1$  coins.

This recurrence relation can be solved using either a bottom-up or top-down approach. The bottom-up approach is done by filling a table, while the top-down approach is done by using recursion. Both approach have the same time complexity which is  $O(n)$

In the end, the maximum value obtained from the coin-row problem can be obtained from  $F(n)$ , which represents the maximum value that can be obtained from the first  $n$  coins.