Class 20

DYNAMIC PROGRAMMING

Logistics

Please arrive for class ON TIME Monday (and everyday)

Convocation Schedule on Monday for MLK Day

Regular Meeting Time		Meeting time on 1/16
8:00 – 8:50	→	8:00 – 8:40
9:00 – 9:50	→	8:50 – 9:30
10:00 – 10:50	→	9:40 – 10:20
11:00 – 11:50	→	10:30 – 11:10
None	→	Convocation Period
12:00 – 12:50	→	1:00 – 1:40
1:00 – 1:50	→	1:50 – 2:30
2:00 – 2:50	→	2:40 – 3:20
3:00 – 3:50	→	3:30 – 4:10
4:00 – 4:50	→	4:20 – 5:00

Logistics

Exam 2 on Thursday, January 19

- 7 - 8:30pm
- Section 01 - Crapo G219
- Section 02 - Crapo G220
- Covers material from December 12 through January 12: Section 4.5, Chapters 5, 6, and 7
- You may use a 1-page cheat sheet during the exam

No class next Friday, January 20 in exchange for evening exam

Next week is advising week – make sure you discuss your registration plan with them!

Dynamic Programming

Technique for solving problems with overlapping sub-problems.

Sub-problems arise from a recurrence relating a given problem's solution to solutions of its smaller sub-problems.

Rather than solving overlapping sub-problems over and over again, solve each of the smaller sub-problems once and record solutions in a table.

Next time, look up solutions in the table.

Invented by American mathematician Richard Bellman in the 1950s to solve optimization problems

- "Programming" here means "planning," not computer programming

Example: Fibonacci

Problem: compute the *n*th Fibonacci number
Recursive definition: $f_0=0$, $f_1=1$, $f_n = f_{n-1} + f_{n-2}$.
First several: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, …

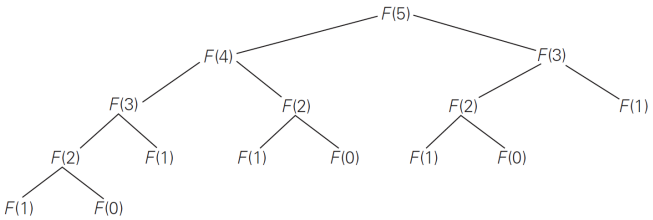

FIGURE 2.6 Tree of recursive calls for computing the 5th Fibonacci number by the definition-based algorithm.

Example: Fibonacci

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Recursive definition: $f_0=0$, $f_1=1$, $f_n = f_{n-1} + f_{n-2}$.
First several: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, …

```
private static long fib_array_helper(int n, long[] fibs) {
   if (n == 1) return fibs[1];
   if (n == 2) return fibs[2];
   if (fibs[n] == 0) fibs[n] = fib_array_helper(n-1, fibs) +
                               fib_array_helper(n-2, fibs);
   return fibs[n];
}
```

Running time is Θ(n)

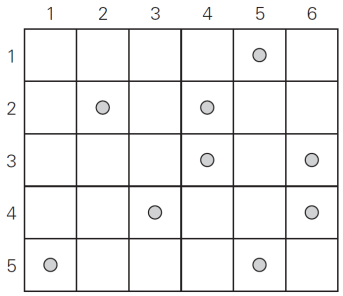
Robot Coin Collection

Coins are placed on an *n* x *m* board.
At most one coin per cell.

Robot:

- starts out in <1,1>
- Deposit at cell <*n,m*>
- On each step, either move:
  - down one cell, or
  - right one cell
- When robot visits a cell with a coin, it automatically gets picked up

Collect maximum # of coins.



Robot Coin Collection

Let *F*(*i, j*) be the largest number of coins the robot can collect and bring to *cell*(*i, j*).

It can reach that cell either from *cell*(*i-1, j*) above or from *cell*(*i, j-1*) to the left.

The largest number of coins that can be brought to those cells are *F*(*i-1, j*) and *F*(*i-1, j*), respectively.

The largest number of coins a robot can bring to *cell*(*i, j*) is the maximum of those two numbers, plus one possible coin at *cell*(*i, j*):

$$F(i, j) = \max\{F(i-1, j), F(i, j-1)\} + c_{ij} \quad \text{for } 1 \le i \le n, \quad 1 \le j \le m$$
$$F(0, j) = 0 \text{ for } 1 \le j \le m \quad \text{and} \quad F(i, 0) = 0 \text{ for } 1 \le i \le n,$$

Robot Coin Collection

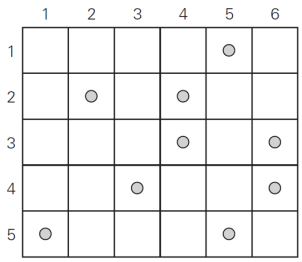
Using the DP formulation, we can fill in an *n* x *m* table for *F*(*i, j*).

ALGORITHM *RobotCoinCollection*$(C[1..n, 1..m])$

```
//Applies dynamic programming to compute the largest number of
//coins a robot can collect on an n × m board by starting at (1, 1)
//and moving right and down from upper left to down right corner
//Input: Matrix C[1..n, 1..m] whose elements are equal to 1 and 0
//for cells with and without a coin, respectively
//Output: Largest number of coins the robot can bring to cell (n, m)
F[1, 1] ← C[1, 1];   for j ← 2 to m do F[1, j] ← F[1, j − 1] + C[1, j]
for i ← 2 to n do
    F[i, 1] ← F[i − 1, 1] + C[i, 1]
    for j ← 2 to m do
        F[i, j] ← max(F[i − 1, j], F[i, j − 1]) + C[i, j]
return F[n, m]
```

Robot Coin Collection

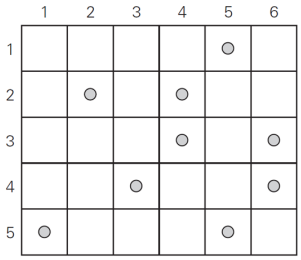
DP results for the problem posted earlier:



	1	2	3	4	5	6
1	0	0	0	0	1	1
2	0	1	1	2	2	2
3	0	1	1	3	3	4
4	0	1	2	3	3	5
5	1	1	2	3	4	**5**

Robot Coin Collection

What if we need the path, in addition to the final coin count?

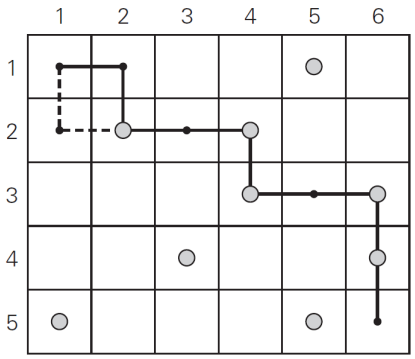


	1	2	3	4	5	6
1	0	0	0	0	1	1
2	0	1	1	2	2	2
3	0	1	1	3	3	4
4	0	1	2	3	3	5
5	1	1	2	3	4	**5**

Robot Coin Collection

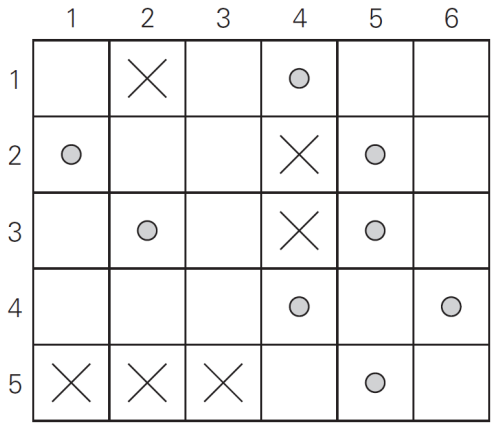
Two optimal paths:

	1	2	3	4	5	6
1	0	0	0	0	1	1
2	0	1	1	2	2	2
3	0	1	1	3	3	4
4	0	1	2	3	3	5
5	1	1	2	3	4	**5**

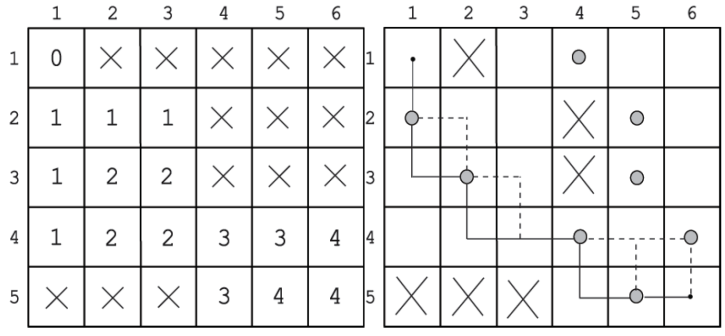


Robot Coin Collection Modified

How do we need to modify our algorithm if some cells are inaccessible to the robot?



Robot Coin Collection Modified



Maximum of 4 coins, 12 possible paths.

Problem Types Where DP Excels

Optimization problems: "minimize …" or "maximize …"

Counting problems: "count the number of ways…"

Often only need the optimal/total value. If also need the solution itself, can *backtrack*

Midterm Reflection for CSSE/MA 473

Please fill in the form distributed in class to hand in.

Honest answers will help you succeed in the course!

Logistics

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