

# CSSE 473 - Winter 2022/23 - Worksheet 1

## Algorithm Definition

1. Greatest common divisor (GCD) problem: given integers  $a, b \geq 0$  (not both zero), find the greatest common divisor  $d$ , i.e.  $\max\{d : d \mid a, d \mid b\}$ .

Which of the following are algorithms for the GCD problem?

- (a) ALGORITHM(?)  $\gcd(a, b)$  by the Fundamental Theorem of Arithmetic
  - 1) Find the prime factors of  $a$ .
  - 2) Find the prime factors of  $b$ .
  - 3) Identify all the common factors (with multiplicity) in the two prime expansions found in steps 1 & 2.
  - 4) Compute the product of all the common factors and return this as the answer.
  
- (b) ALGORITHM(?)  $\gcd(a, b)$  by consecutive integer checking
  - 1) Assign the value of  $\min\{a, b\}$  to  $t$ .
  - 2) Divide  $a$  by  $t$ . If the remainder is 0, go to step 3. Otherwise, go to step 4.
  - 3) Divide  $b$  by  $t$ . If the remainder is 0, return the value of  $t$  as the answer and stop. Otherwise, go to step 4.
  - 4) Decrease the value of  $t$  by 1. Go to Step 2.
  
- (c) ALGORITHM(?)  $\gcd(a, b)$  by coprime assumption
  - 1) Return 1.
  
- (d) ALGORITHM(?)  $\gcd(a, b)$  by Euclid's algorithm

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while  $b \neq 0$ :
     $r \leftarrow a \bmod b$ 
     $a \leftarrow b$ 
     $b \leftarrow r$ 
return  $a$ 
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