Relative Motion

This is one of the most important topics in this course.

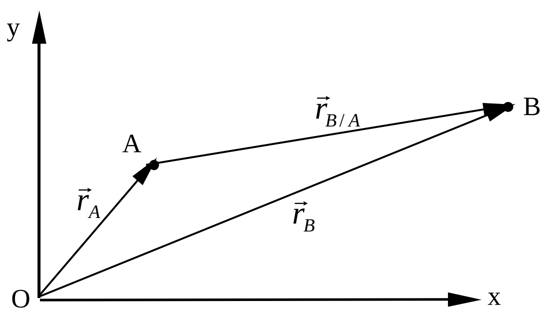
All motion is relative. We have to have some frame of reference (coordinate system) in order to measure position, velocity and acceleration.

For this course, these frames of reference can be put into two groups. First, reference points that are fixed, usually with respect to the earth or the laboratory. I know these frames of reference are moving, with respect to say the sun or the stars, but they don't have much acceleration to speak of. It's usually safe to think of them as "fixed."

The second type of reference is going to be discussed at length in these notes. It is a reference system that is itself moving.

Let's say that observer A is at the origin of a moving reference frame. He or she observes the motion of another point B. A could measure a position, velocity and acceleration for B. But these motions are not the same that an observer in the fixed frame would measure by observing B. The difference would be A's motion.

Here are the basic equations, which stem from the simple drawing.



O = origin of fixed coordinate system
A = origin of moving coordinate system
B = another moving point

Note how we find that to get to point B we can either go there straight ahead, or we can go first to A and then head over to B. This idea can be expressed vectorially as follows:

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A} \qquad (1)$$

In words, Eq. (1) is: the position of B is equal to the position of A plus the relative position of B with respect to A. Now let's differentiate the previous equation with respect to time. What we obtain is a similar statement about the velocities.

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A} \qquad (2)$$

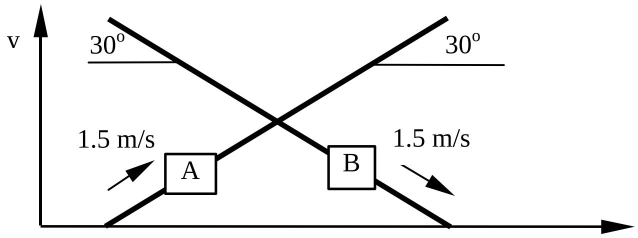
or in words: the velocity of B is equal to the velocity of A plus the relative velocity of B with respect to A. Now differentiate with respect to time again.

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A} \qquad (3)$$

This equation says the acceleration of A plus the acceleration of B as seen from A is equal to the acceleration of B measured in the same fixed frame that A's position is measured in.

For now, let's not let the moving frame rotate. This is a topic we will investigate in the future

Known: There are two escalators, as shown in the figure.
Find: Determine the relative velocity of B with respect to A.
Given: A is ascending at a constant 1.5 m/s and B is descending at a constant 1.5 m/s.

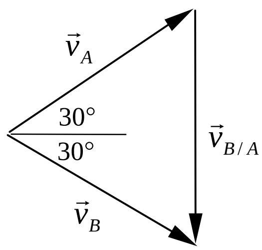


Analysis :
Strategy: Use relative motion equations

We know $$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

We have two approaches at this point, 1) vector diagram and 2) vector algebra.

Vector diagram approach:



$\vec{v}_A$ =1.5 m/s

$\vec{v}_B$ =1.5 m/s

Clearly from the diagram and the angles given in the problem this is an equilateral triangle so $\vec{v}_{B/A} = -1.5\hat{j}\,\mathrm{m/s}$

Vector algebra approach: Write all the velocity in terms of their components and then equate components.

$$\vec{v}_B = 1.5\cos(30^\circ)\hat{i} - 1.5\sin(30^\circ)\hat{j}$$
$$\vec{v}_A = 1.5\cos(30^\circ)\hat{i} + 1.5\sin(30^\circ)\hat{j}$$
$$\vec{v}_{B/A} = v_{B/A_x}\hat{i} + v_{B/A_y}\hat{j}$$

substituting into the relative velocity equation we get:

$$1.5\cos(30^\circ)\hat{i} - 1.5\sin(30^\circ)\hat{j} = 1.5\cos(30^\circ)\hat{i} + 1.5\sin(30^\circ)\hat{j} + v_{B/A_x}\hat{i} + v_{B/A_y}\hat{j}$$

equating components gives:

$$\hat{i}: \quad 1.5\cos(30^\circ) = 1.5\cos(30^\circ) + v_{B/A_x} \longrightarrow v_{B/A_x} = 0\, m/s$$
$$\hat{j}: \quad -1.5\sin(30^\circ) = 1.5\sin(30^\circ) + v_{B/A_y} \longrightarrow v_{B/A_y} = -1.5\, m/s$$
$$v_{B/A} = 0\hat{i} - 1.5\hat{j}\; m/s$$

See also examples 3/2 and 3/3 in the text.