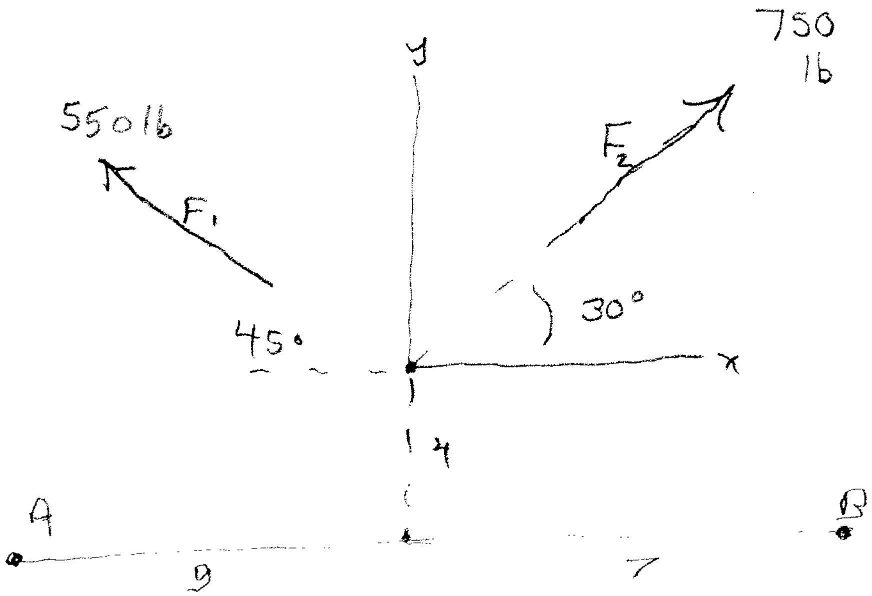
5.15



$M = r_x F_y - r_y F_x$

(a) Moment of F_1 about A

$M_A = 9(550 \sin 45°) - 4(-550 \cos 45°)$

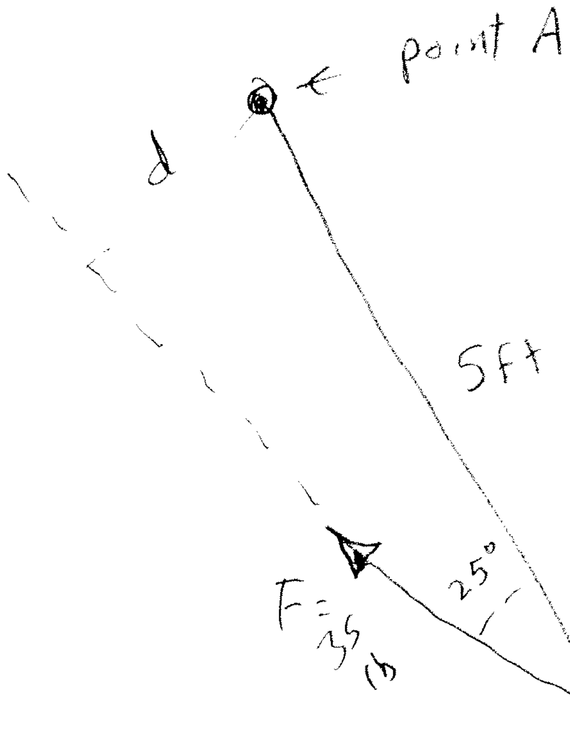
$= 5056 = \boxed{5060 \text{ in-lb ccw}}$

(b) Moment of F_2 about B

$M_B = (-7)(750 \sin 30°) - 4(750 \cos 30°)$

$= -5223 = \boxed{-5220 \text{ in-lb cw}}$

5.17



$d = 5 \sin 25° = 2.113 \text{ ft}$

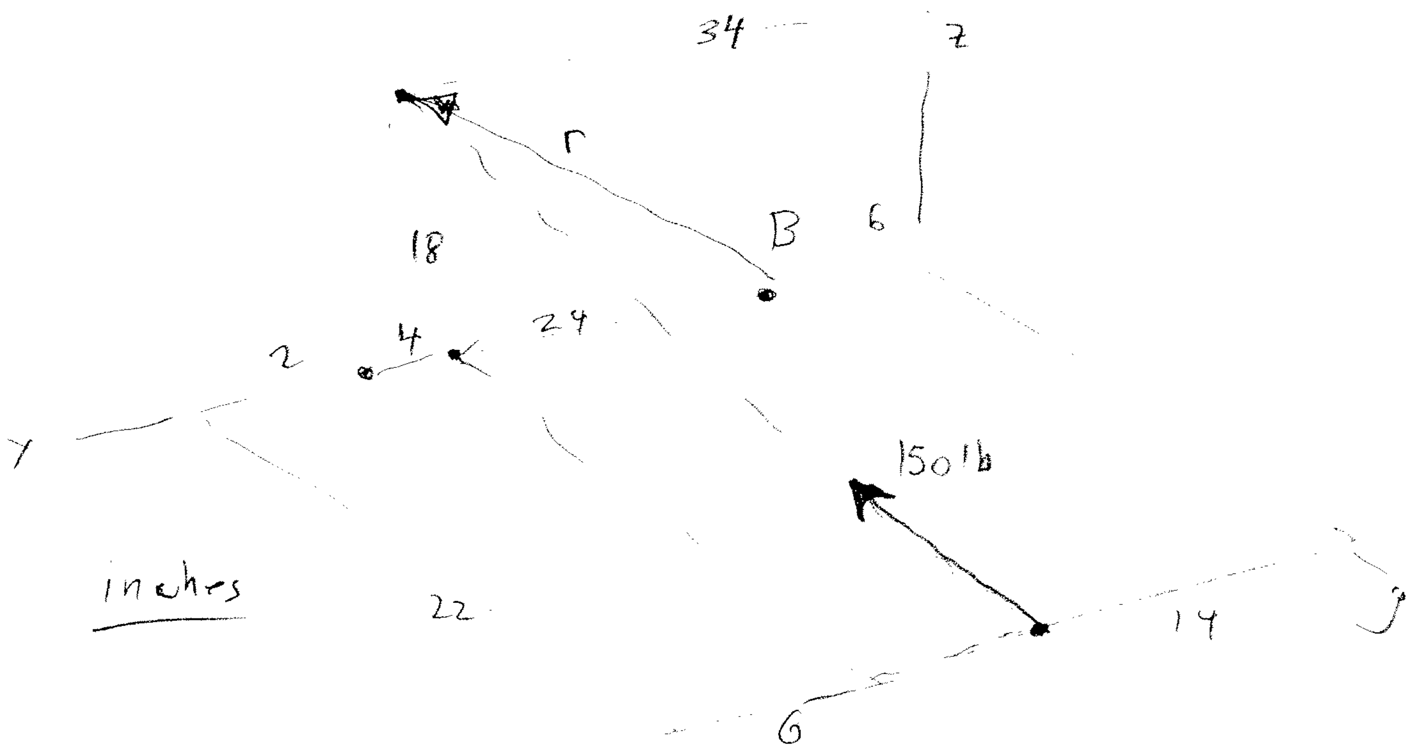
$M = F d = (35 \text{ lb})(2.113 \text{ ft})$

$= \boxed{74.0 \text{ ft-lb}}$ CW

5.23

$d = \sqrt{20^2 + 22^2 + 18^2}$ $\quad d_z = \frac{18}{d}$

$d_x = 16/d \quad d_y = \frac{-22}{d}$



$\vec{T} = 150\,(d_x \vec{i} + d_y \vec{j} + d_z \vec{k})$

$= 86.315\,\vec{i} - 94.947\,\vec{j} + 77.684\,\vec{k}$

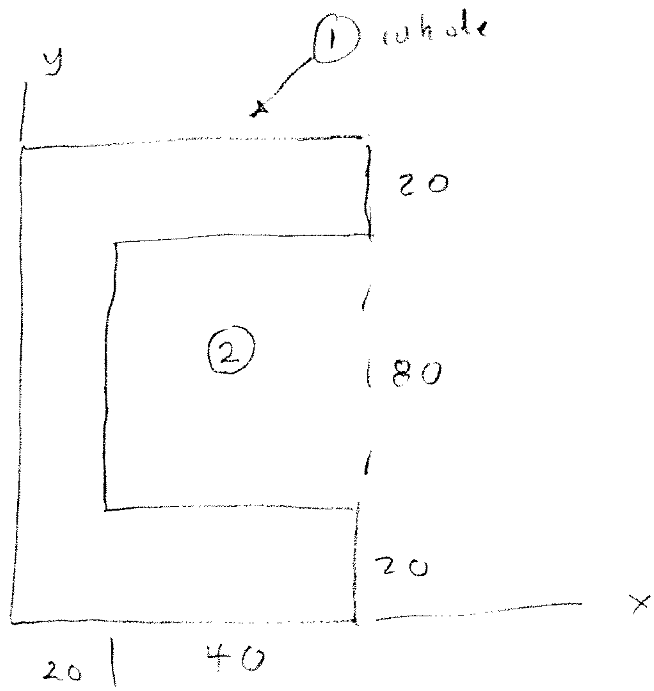
$\vec{r}_B = 28\,\vec{i} + 0\,\vec{j} + 18\,\vec{k} \qquad r_c = 8\vec{i} + 22\vec{j}$

$\vec{M}_B = \left\{\begin{matrix}\vec{r}_B \\ r_c\end{matrix}\right\} \times \vec{T} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \left\{\begin{matrix}28 \\ 8\end{matrix}\right\} & \left\{\begin{matrix}0 \\ 22\end{matrix}\right\} & \left\{\begin{matrix}18 \\ 0\end{matrix}\right\} \\ 86.315 & -94.947 & 77.684 \end{vmatrix}$

$= \underline{1709\,\vec{i} - 621\,\vec{j} - 2659\,\vec{k} \text{ in-lb}}$

for both!

5-94



By symm. $\bar{y} = 60$

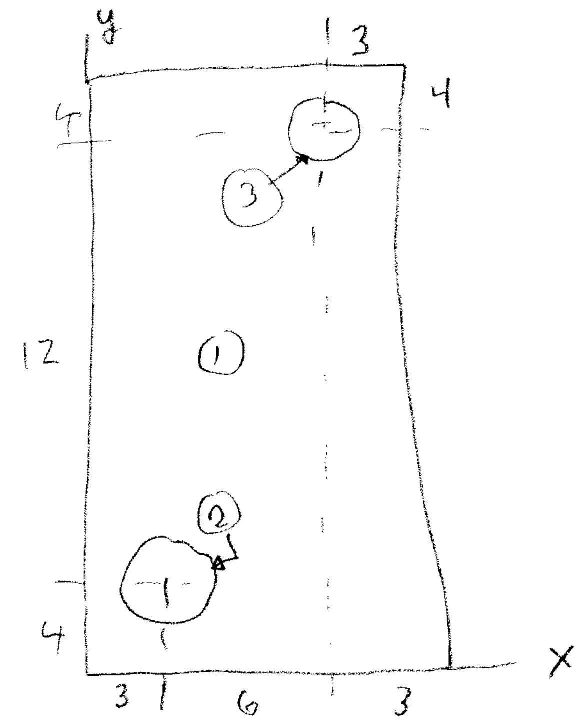
Use the cut out approach

$$\bar{x} = \frac{\sum x_i A_i}{\sum A_i} = \frac{[(60)(120)]30 - [40(80)]40}{(60)(120) - (40)(80)}$$

$$= \frac{88000}{4000} = 22$$

$$\boxed{\begin{aligned} \bar{x} &= 22 \text{ mm} \\ \bar{y} &= 60 \text{ mm} \end{aligned}}$$

5-95



Due to the symm dist of holes

one might argue

$\bar{x} = 6$
$\bar{y} = 10$

Let us see if this is true.

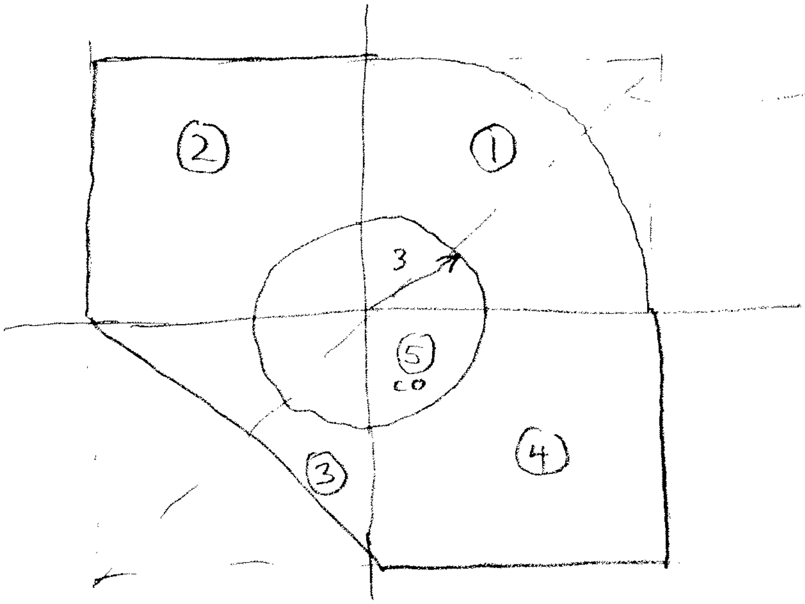
Fig	Area	x	y	xA	yA
1	240	6	10	1440	2400
2	-4π	3	4	-12π	-16π
3	-4π	9	16	-36π	-64π
	$240-8\pi$			$1440-48\pi$	$2400-80\pi$

$$\bar{x} = \frac{1440-48\pi}{240-8\pi} = 6$$

$$\bar{y} = \frac{2400-80\pi}{240-8\pi} = 10$$

$$\boxed{\begin{aligned}\bar{x} &= 6\text{in}\\ \bar{y} &= 10\text{in}\end{aligned}}$$

5-99

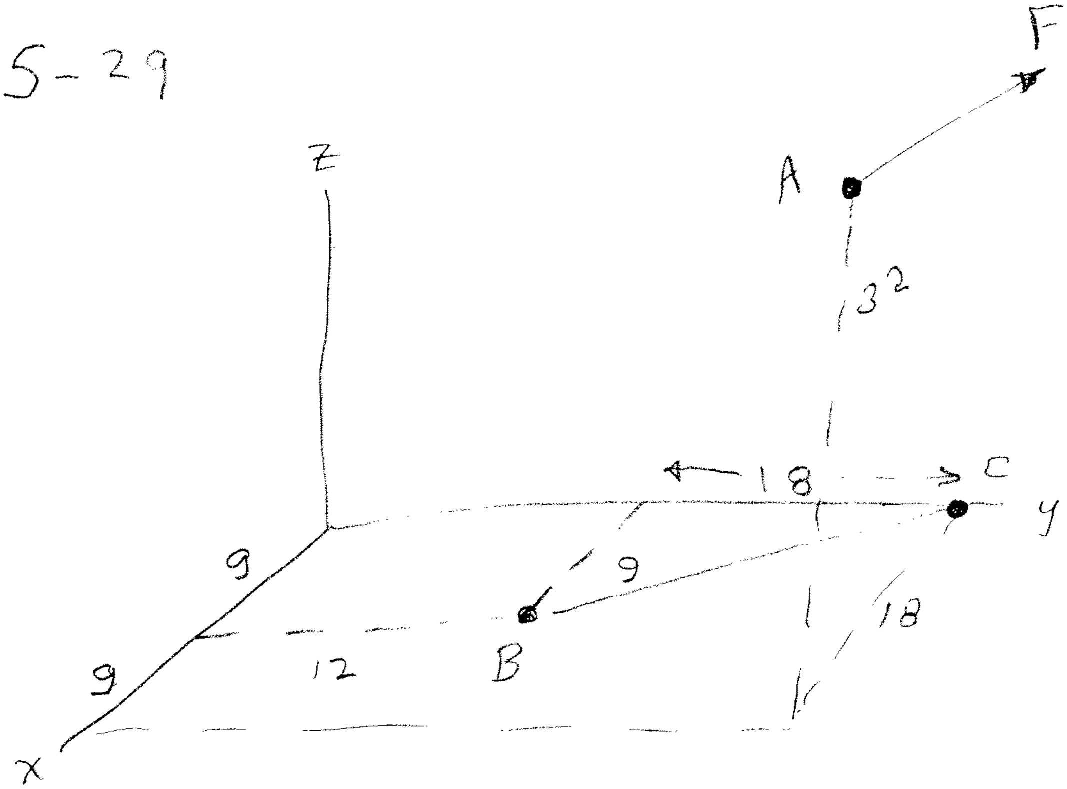


you can assume it will lie on this line

Figure	Area	x	y	xA	yA
1	38.48451	2.9708923	2.97089227	114.3333333	114.333333
2	49	-3.5	3.5	-171.5	171.5
3	24.5	-2.333333	-2.3333333	-57.1666667	-57.1666667
4	49	3.5	-3.5	171.5	-171.5
5	-28.2743	0	0	0	0
Sum	132.7102			57.16666667	57.1666667

xbar	0.4307632
ybar	0.4307632

5-29



$$\vec{r}_{BA} = 9\vec{i} + 18\vec{j} + 32\vec{k} \quad (\text{in})$$

$$\vec{F} = 60\vec{i} + 100\vec{j} + 120\vec{k} \quad (\text{lb})$$

$$\vec{r}_{BA} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 9 & 18 & 32 \\ 60 & 100 & 120 \end{vmatrix} = -1040\vec{i} + 840\vec{j} - 180\vec{k}$$

in-lb

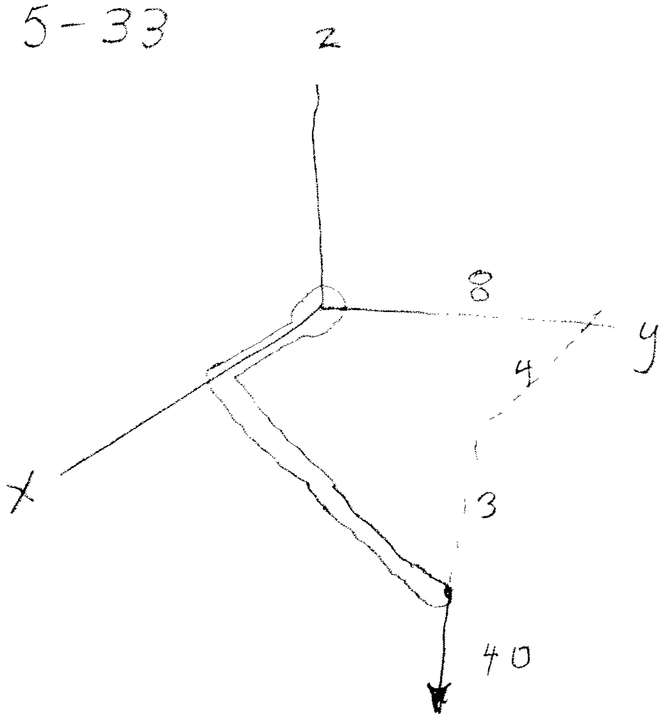
$$\vec{n}_{BC} = \frac{-9\vec{i} + 18\vec{j} + 0\vec{k}}{\sqrt{(-9)^2 + 18^2}} = -.4472\vec{i} + .8944\vec{j}$$

$$M_{BC} = \vec{n}_{BC} \cdot (\vec{r}_{BA} \times \vec{F}) = \underline{1216 \text{ in-lb}}$$

```
>> r

r =

     9    18    32

>> F

F =

    60   100   120

>> M = cross(r,F)

M =

       -1040          840         -180

>> nBC = 1/sqrt(9^2+18^2) * [ -9 18 0]

nBC =

   -0.4472    0.8944         0

>> MBC = dot(nBC,M)

MBC =
```

```
   1.2164e+003

>>
```

5-33



$$\vec{r} = 4\vec{i} + 8\vec{j} - 3\vec{k}$$
$$\vec{F} = -40\vec{k}$$

$$\vec{r} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 8 & -3 \\ 0 & 0 & -40 \end{vmatrix}$$

$$= \vec{i}\left[(8)(-40) - (0)(-3)\right]$$
$$- \vec{j}\left[(4)(-40) - (0)(-3)\right]$$
$$+ \vec{k}\left[(4)(0) - (0)(8)\right]$$

$$= -320\vec{i} + 160\vec{j}$$

Unit vector along axis:

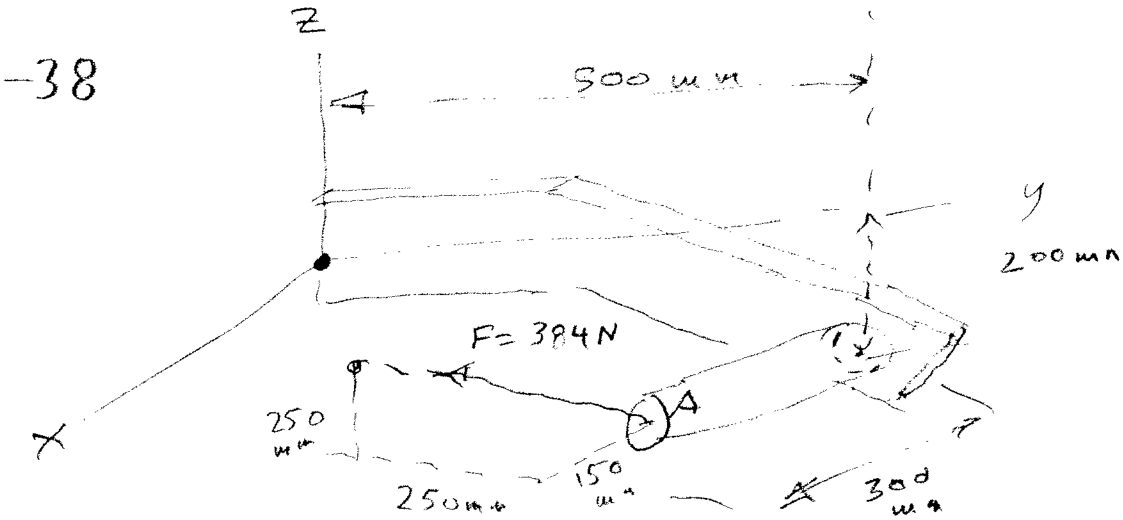
$$\vec{n} = \vec{i}$$

$$M_{axis} = M_x = \vec{i} \cdot (-320\vec{i} + 160\vec{j})$$

$$= -320 \text{ in-lb}$$

Magnitude is $\boxed{320}$ in-lb

5-38



O is a good ref pt since it lies on all axes mentioned.

$$\vec{r}_{OA} = 300\vec{i} + 500\vec{j} - 200\vec{k}$$

$$\vec{F} = \frac{384}{\sqrt{150^2+250^2+250^2}}(150\vec{i} - 250\vec{j} + 250\vec{k})$$

$$= 150\vec{i} - 250\vec{j} + 250\vec{k} \text{ N}$$

$$\vec{M}_0 = \vec{r}_{OA} \times \vec{F} = 74{,}990\vec{i} - 104{,}980\vec{j} - 149{,}980\vec{k}$$

For twist about y: $M_y = \vec{j} \cdot \vec{M}_0 = -104{,}980$ N-mm

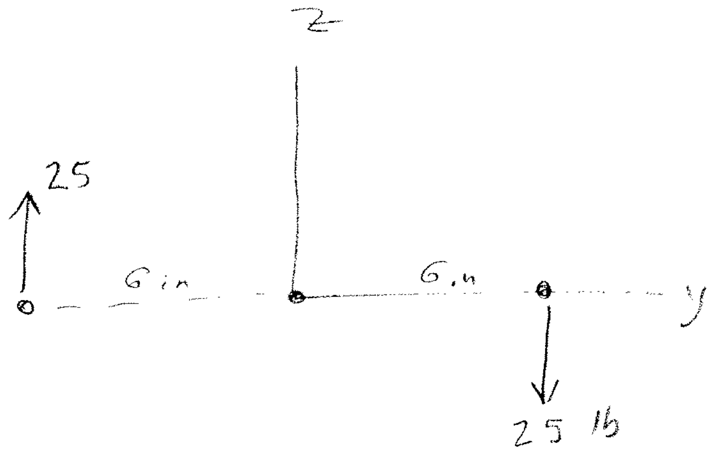
$= \underline{-1050 \text{ N-m}}$

For bend about x $\quad M_x = \vec{i} \cdot M_0$

$M_x = \underline{75.0 \text{ N}\cdot\text{m}}$

```
>> rOA

rOA =

   300   500  -200

>> F

F =

  149.9776 -249.9627  249.9627

>> M = cross(rOA,F)

M =

  1.0e+005 *

    0.7499   -1.0498   -1.4998

>>
```

5-43



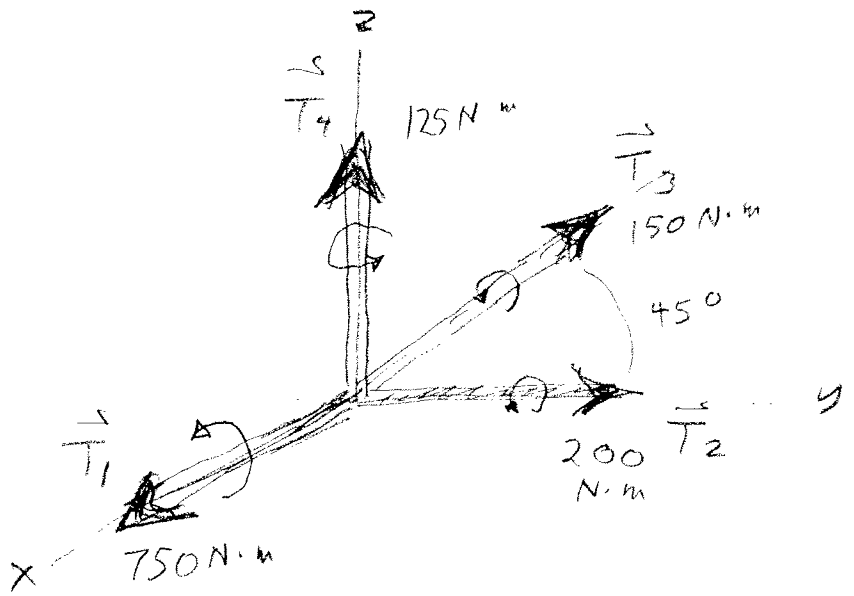
$$M = Fd = (25\,lb)(12\,in)$$

$$= 300\ in\text{-}lb\ \ \underline{cw}$$

So in cartesian vector form

$$\vec{M} = \underline{-300\vec{i}} + 0\vec{j} + 0\vec{k}\ \ in\text{-}lb$$

5-46



$$\vec{T} = \vec{T}_1 + \vec{T}_2 + \vec{T}_3 + \vec{T}_4$$

$$= 750\vec{i} + 200\vec{j} + 150\,(\cos 45^\circ \vec{j} + \sin 45^\circ \vec{k}) + 125\,\vec{k}$$

$$= 750\vec{i} + 306.06\,\vec{j} + 231.066\,\vec{k}$$

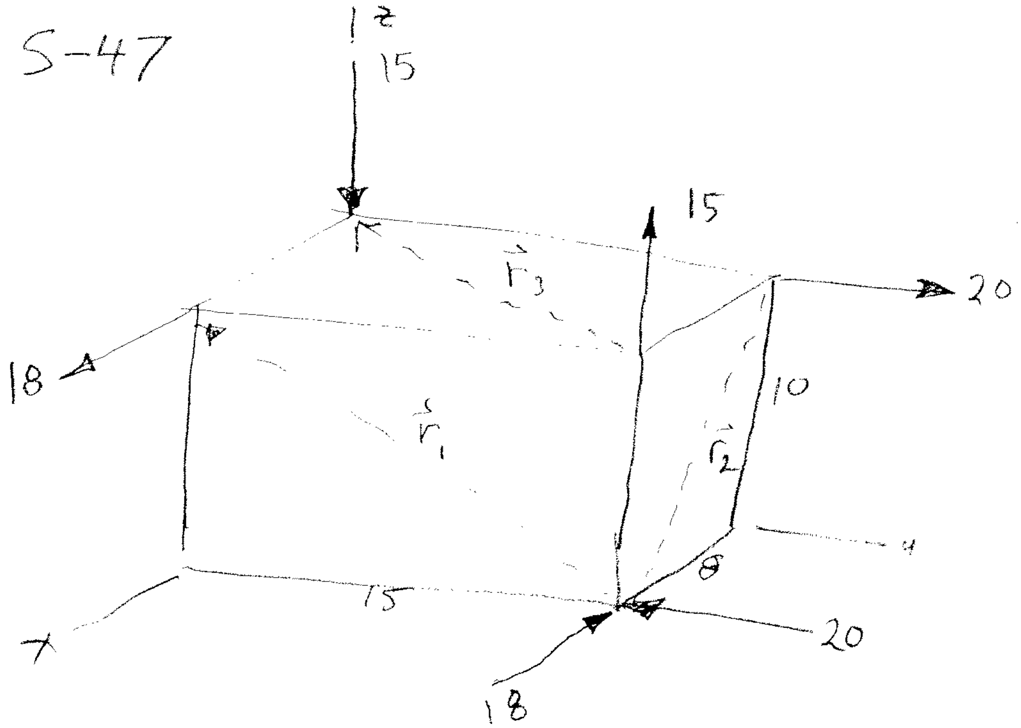
$$T = \sqrt{750^2 + 306.06^2 + 231.066^2} = \underline{842 \text{ N·m}}$$

direction given by unit vector

$$\vec{n} = \frac{\vec{T}}{T} = 0.8904\,\vec{i} + 0.3633\,\vec{j} + 0.2743\,\vec{k}$$

$$\theta_x = \cos^{-1}(0.8904) = 27.1^\circ$$
$$\theta_y = \cos^{-1}(.3633) = 68.7^\circ$$
$$\theta_z = \cos^{-1}(.2743) = 74.1^\circ$$

S-47



$$\vec{M}_1 = (0\vec{i} - 15\vec{j} + 10\vec{k}) \times 18\vec{i} = 180\vec{j} + 270\vec{k}$$

$$\vec{M}_2 = (-8\vec{i} + 0\vec{j} + 10\vec{k}) \times 20\vec{j} = -200\vec{i} - 160\vec{k}$$

$$\vec{M}_3 = (-8\vec{i} - 15\vec{j} + 0\vec{k}) \times -15\vec{k} = 225\vec{i} - 120\vec{j}$$

$$\vec{M} = \vec{M}_1 + \vec{M}_2 + \vec{M}_3 = 25\vec{i} + 60\vec{j} + 110\vec{k} \text{ in-lb}$$

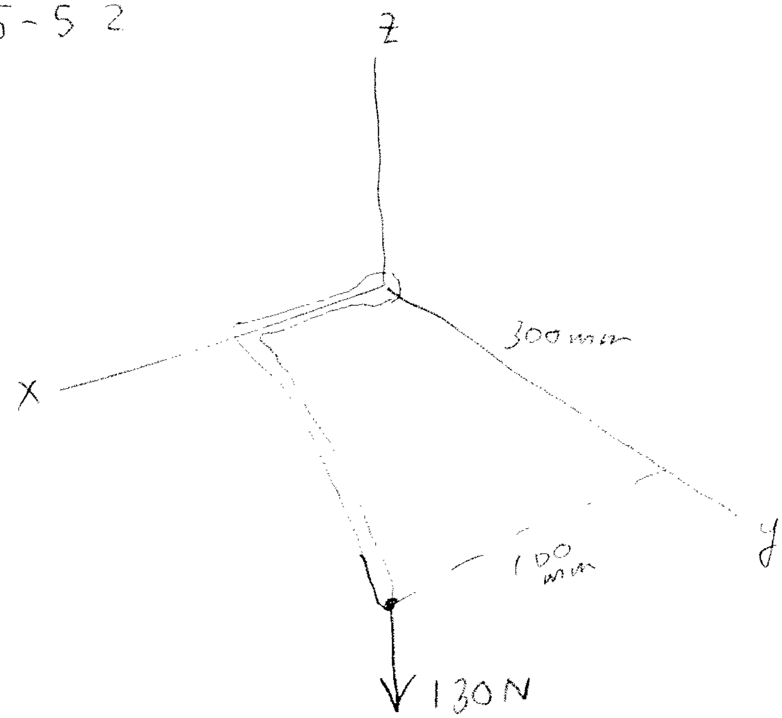
$$M = \sqrt{25^2 + 60^2 + 110^2} = \underline{\underline{127.8 \text{ in-lb}}}$$

$$\theta_x = \cos^{-1}\left(\frac{25}{127.8}\right) = 78.7^\circ$$

$$\theta_y = \cos^{-1}\left(\frac{60}{127.8}\right) = 62.0^\circ$$

$$\theta_z = \cos^{-1}\left(\frac{110}{127.8}\right) = 30.6^\circ$$

5-5 2



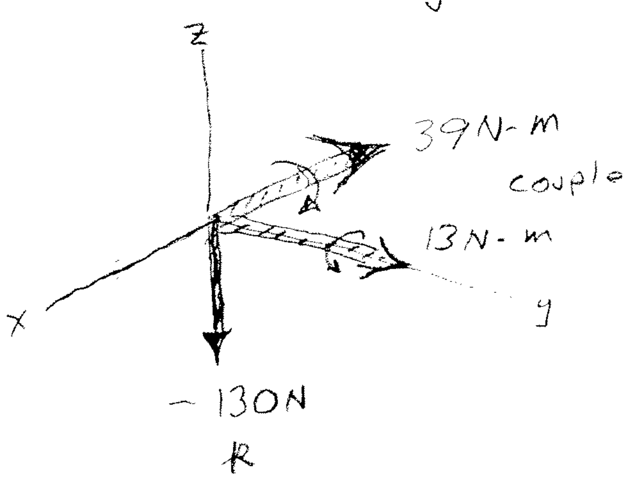
$\vec{F} = -130\vec{k}$

$\vec{r} = 100\vec{i} + 300\vec{j}$

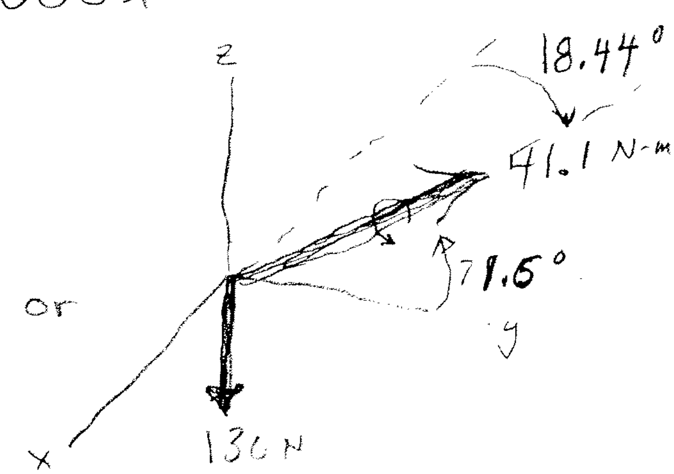
$\vec{M} = \vec{r} \times \vec{F}$

$= (100\vec{i} + 300\vec{j}) \times (-130\vec{k})$

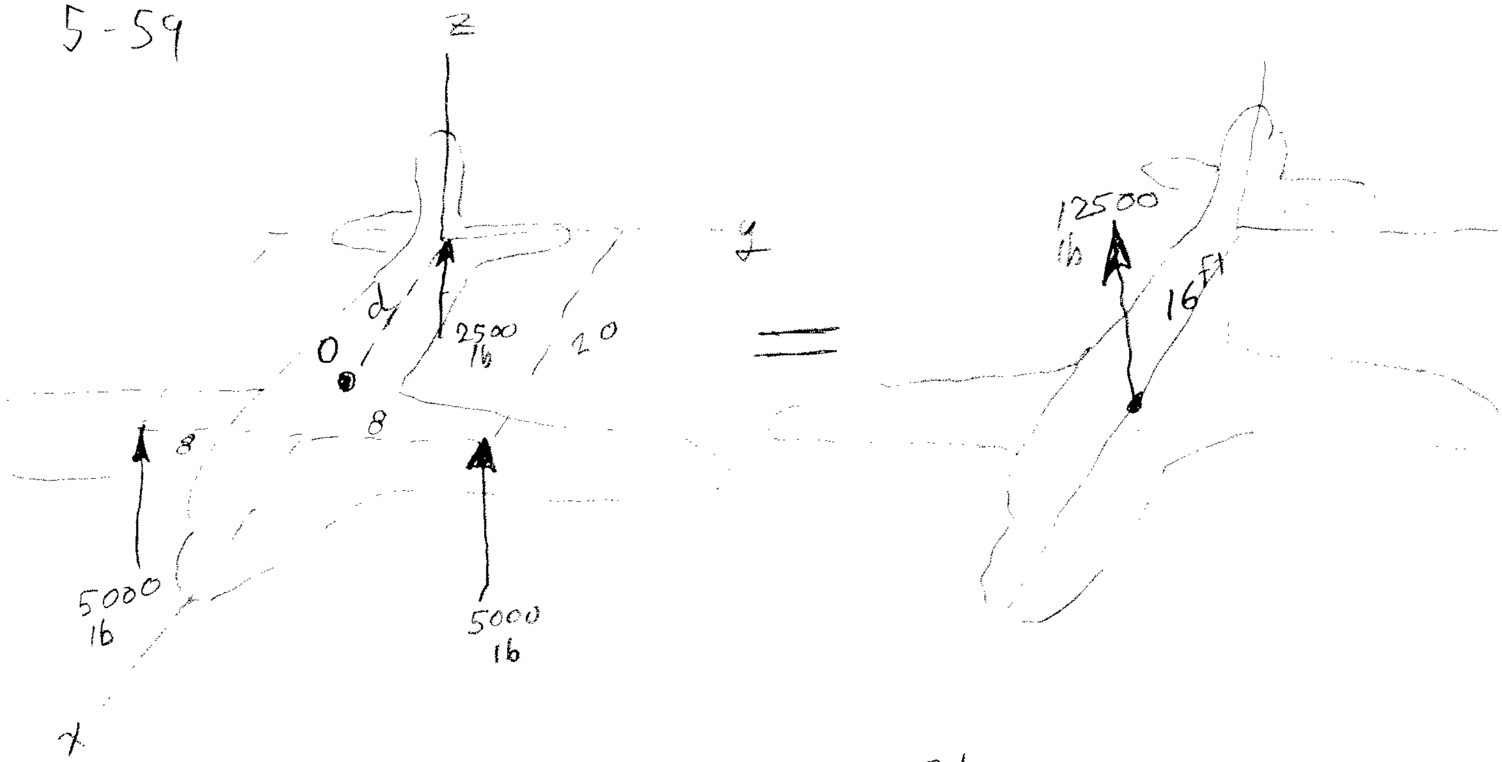
$= 13000\vec{j} - 39000\vec{i}$



or



5-59



Point O is the collection pt.

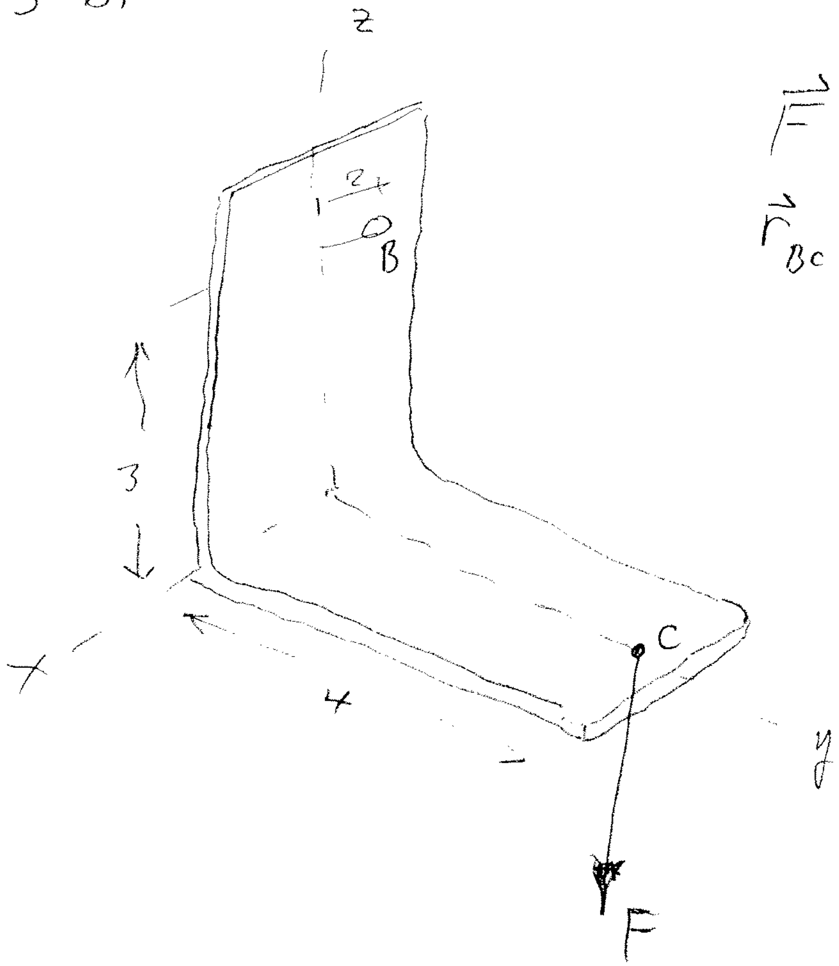
$$\Sigma F_z = 5000 + 5000 + 2500 = 12500 \text{ lb}$$

$$\begin{aligned}\Sigma M_o = & -d\vec{i} \times 2500\vec{k} \\ & + \left[(20-d)\vec{i} + 8\vec{j}\right] \times 5000\vec{k} \\ & + \left[(20-d)\vec{i} - 8\vec{j}\right] \times 5000\vec{k}\end{aligned}$$

$$0\vec{i} + 0\vec{j} + 0\vec{k} = 2500\,d\,\vec{j} - (20-d)(5000)\vec{j} - (20-d)(5000)\vec{j}$$

$\therefore \quad 12500d - 200000 = 0 \qquad d = 16 \text{ ft}$

5-61



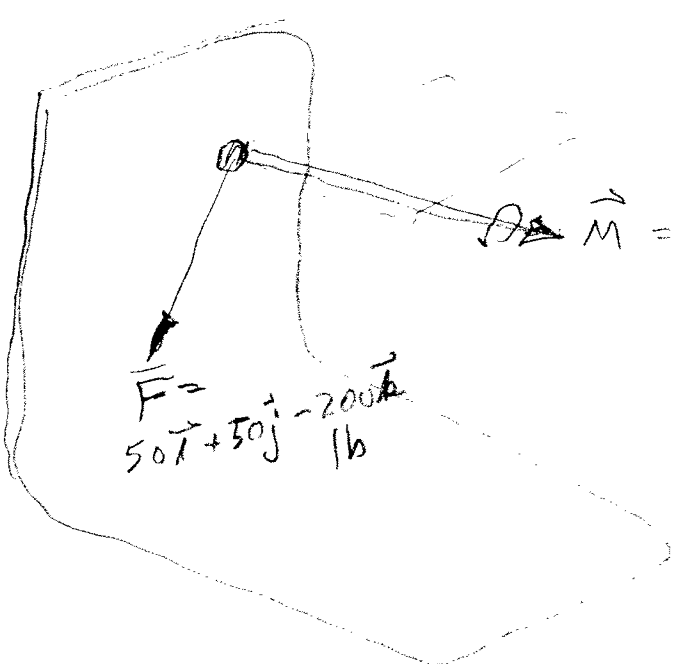
$$\vec{F} = 50\vec{i} + 50\vec{j} - 200\vec{k}$$

$$\vec{r}_{BC} = 2\vec{i} + 4\vec{j} - 3\vec{k}$$

Moment of the couple

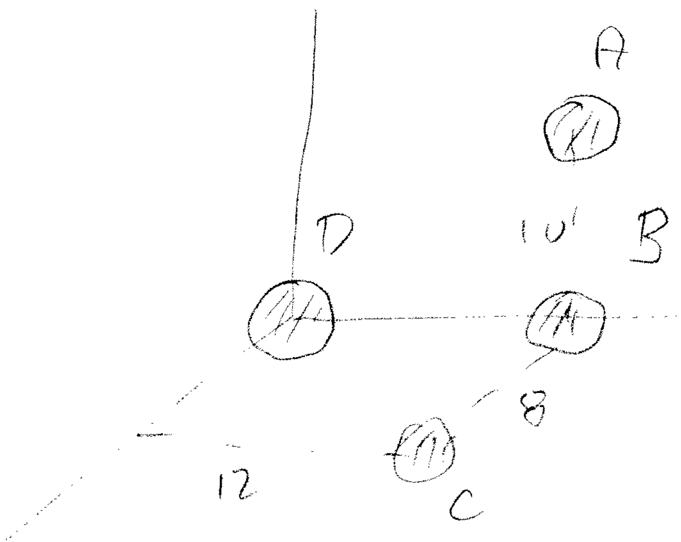
$$\vec{M} = \vec{r}_{BC} \times \vec{F} = -650\vec{i} + 250\vec{j} - 100\vec{k}$$ in-lb

ANS



$\vec{M} = -650\vec{i} + 250\vec{j} - 100\vec{k}$ in lb

5-71



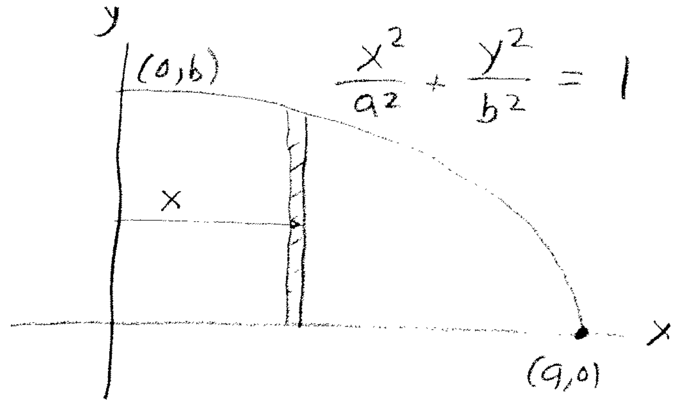
Mass	wt	x	y	z	xW	yW	zW
A	20	0	12	10	0	240	200
B	25	0	12	0	0	300	0
C	30	8	12	0	240	360	0
D	40	0	0	0	0	0	0
Σ	115				240	900	200

$$\bar{x} = \frac{240}{115} = \underline{2.09 \text{ in}}$$

$$\bar{y} = \frac{900}{115} = \underline{7.83 \text{ in}}$$

$$\bar{z} = \frac{200}{115} = \underline{1.739 \text{ in}}$$

5.78



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$y^2 = \left(1 - \frac{x^2}{a^2}\right) b^2$$

$$y = b\left(1 - \frac{x^2}{a^2}\right)^{1/2}$$

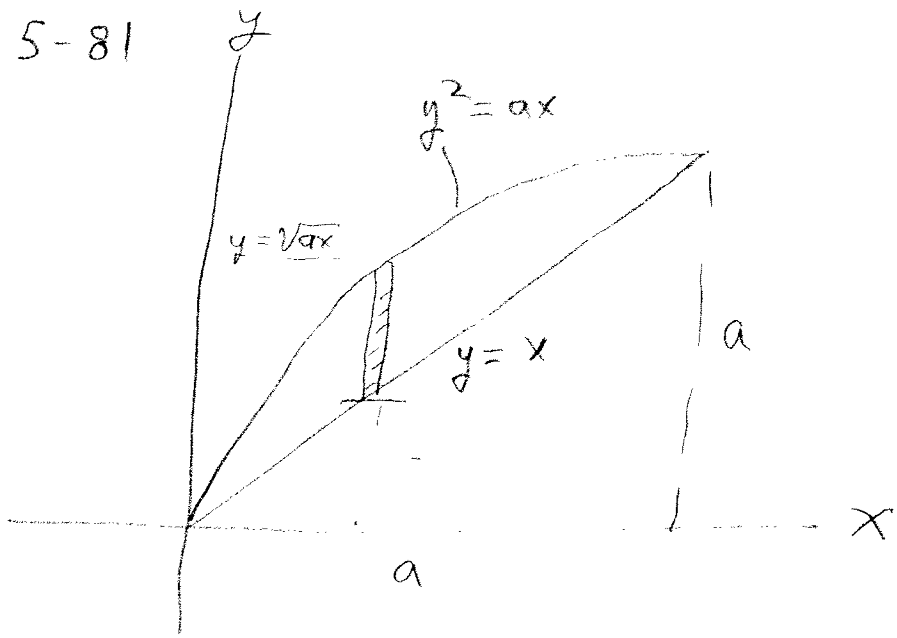
$$dA = y\,dx$$

$$A = \int_0^a b\left(1 - \frac{x^2}{a^2}\right)^{1/2} dx = \frac{\pi a b}{4}$$

$$\bar{x} = \frac{1}{A}\int_0^a x\,b\left(1 - \frac{x^2}{a^2}\right)^{1/2} dx = \frac{\frac{a^2 b}{3}}{\frac{\pi a b}{4}}$$

$$\boxed{\bar{x} = \frac{4a}{3\pi}}$$

5-81



$$dA = \left(\sqrt{ax} - x\right) dx$$

$$A = \int_0^a (\sqrt{ax} - x)\, dx = \frac{a^2}{6}$$

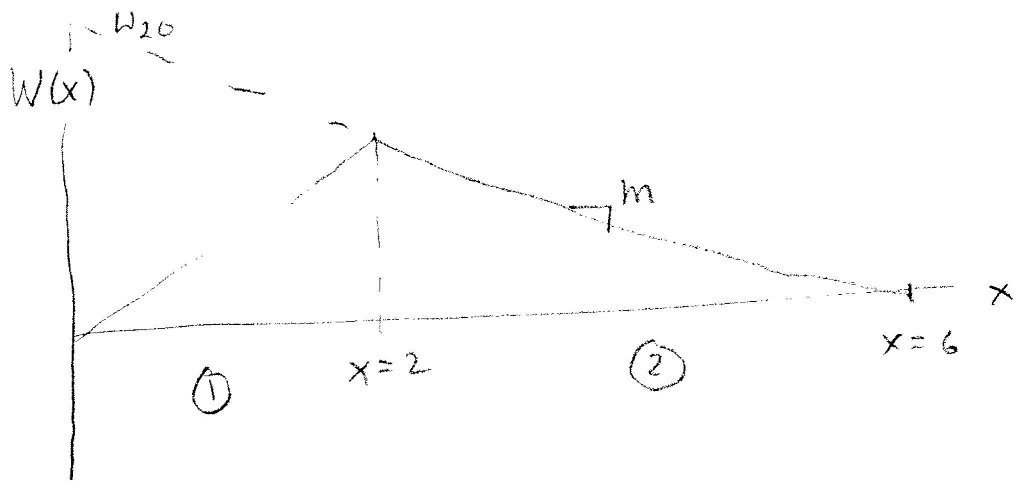
$$A\bar{x} = \int_0^a x(\sqrt{ax} - x)\, dx = \frac{a^3}{15}$$

$$\bar{x} = \frac{a^3/15}{a^2/6} = \frac{2a}{5}$$

$$A\bar{y} = \int_0^a \frac{(x + \sqrt{ax})}{2} \left(\sqrt{ax} - x\right) dx = \frac{a^3}{12}$$

$$\bar{y} = \frac{a}{2}$$

5-108



$w_1 = 375x$

$w_2 = w_{20} - mx$ $\quad w_{20} - 2m = 750$

$w_{20} - 6m = 0$

solve: $w_2 = 1125 - 187.5x$

$A_1 = \int_0^2 375x\,dx = 750$ (area under curve)

$M_1 = \int_0^2 x(375x)\,dx = 1000$

$x_1 = \frac{M_1}{A_1} = 1.333$

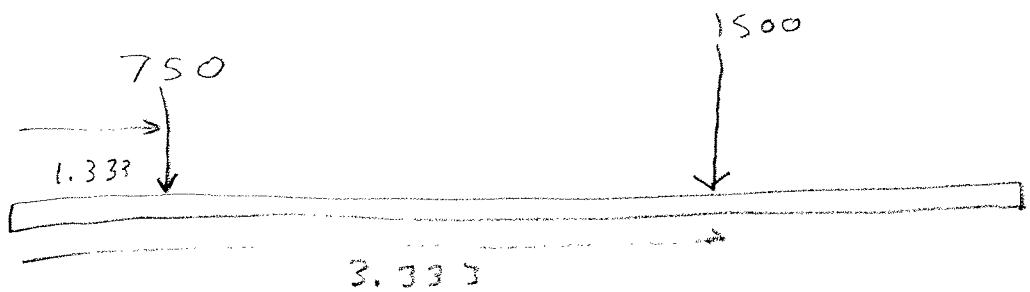
5.108 p2.

$$A_2 = \int_2^6 (1125 - 187.5x)\,dx = 1500$$

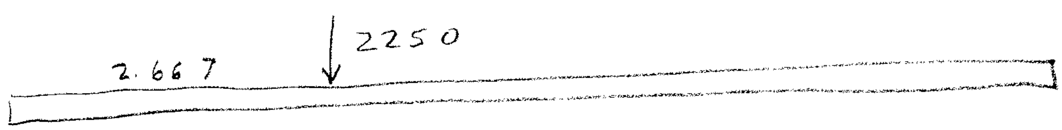
$$\left(= \frac{4 \cdot 750}{2}\right)$$

$$M_2 = \int_2^6 x\,(1125 - 187.5x)\,dx = 5000$$

$$x_2 = 3.333$$



$$\bar{X} = \frac{(1.333)(750) + (3.333)(1500)}{750 + 1500} = 2.667 \text{ m}$$



Answer : 2250 N @ 2.67 m

Maple for solution of 5-108

```
> solve({ w20 - 2*m=750., w20 - 6*m=0},{w20,m});
```

$$\{m = 187.5000000, w20 = 1125.\}$$

```
> w1 := 375*x;
```

$$w1 := 375\,x$$

```
> A1 := int( w1,x=0..2);
```

$$A1 := 750$$

```
> M1 := int( x*w1,x=0..2.);
```

$$M1 := 1000.$$

```
> x1 := M1/A1;
```

$$x1 := 1.333333333$$

```
> w2 := 1125.-187.5*x;
```

$$w2 := 1125. - 187.5\,x$$

```
> A2 := int( w2,x=2..6);
```

$$A2 := 1500.$$

```
> M2 := int( x*w2,x=2..6);
```

$$M2 := 5000.$$

```
> x2 := M2/A2;
```

$$x2 := 3.333333333$$

```
> xbar := (x1*A1+x2*A2)/(A1+A2);
```

$$xbar := 2.666666667$$

Alternative solution with Maple

```
> w := x -> piecewise( x<2, 375*x, 1125-187.5*x);
```

$$w := x \to \text{piecewise}(x < 2, 375\,x, 1125 - 187.5\,x)$$

```
> F := int( w(x), x=0..6);
```

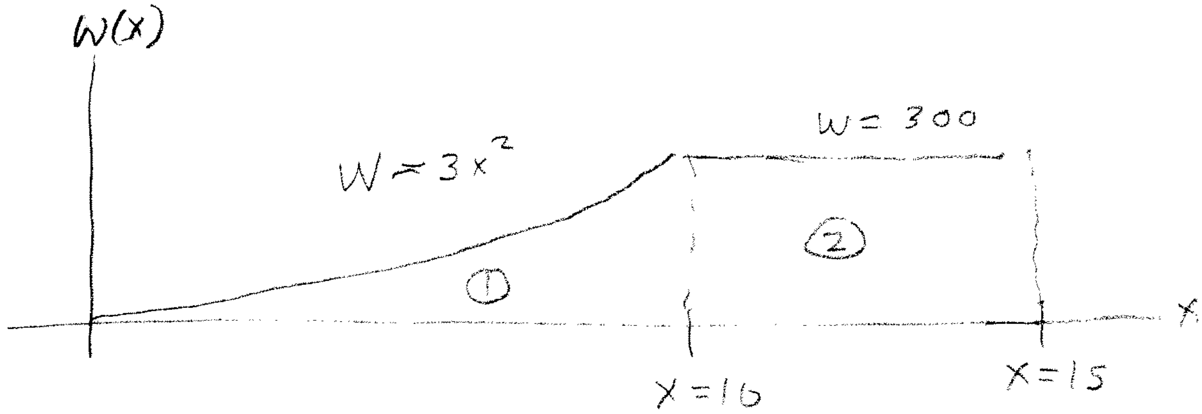
$$F := 2250.$$

```
> xbar := 1/F * int( x*w(x), x=0..6);
```

$$xbar := 2.666666666$$

```
>
```

5 - 111



$w_1 = 3x^2$

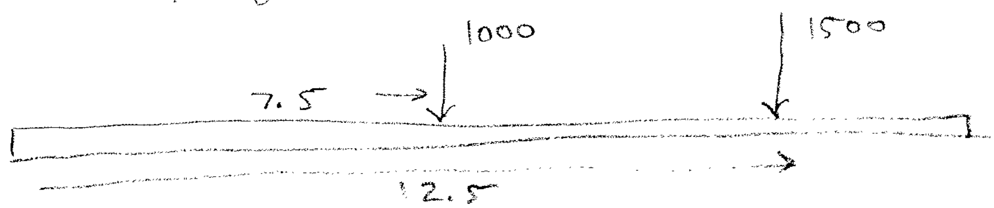
$F_1 = \int_0^{10} w_1 \, dx = 1000$

$x_1 = \frac{1}{F_1} \int_0^{10} x \, w_1 \, dx = 7.5$

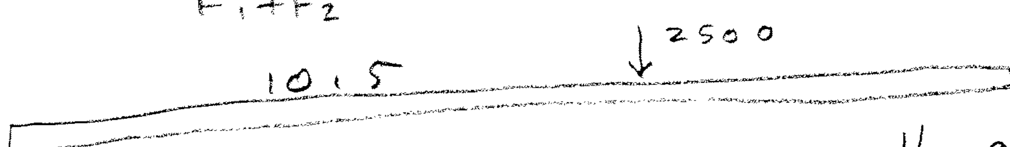
$w_2 = 300$

$F_2 = 1500$

$x_2 = 12.5$



$$\bar{x} = \frac{x_1 F_1 + x_2 F_2}{F_1 + F_2} = \frac{(7.5)(1000) + 12.5(1500)}{2500} = 10.5$$



Answer <u>2500 lb at x = 10.5 f</u>

Maple for 5-111

```
> F1 := int( 3*x^2, x=0..10);
```

$$F1 := 1000$$

```
> x1 := 1/F1 * int( x * 3*x^2, x=0..10.);
```

$$x1 := 7.500000000$$

```
> F2 := 1500.;
```

$$F2 := 1500.$$

```
> x2 := 12.5;
```

$$x2 := 12.5$$

```
> F := F1+F2;
```

$$F := 2500.$$

```
> xbar := 1/F * (x1*F1+x2*F2);
```

$$xbar := 10.50000000$$

Alternate Maple

```
> w :=x-> piecewise( x<10, 3*x^2, 300);
```

$$w := x \rightarrow \mathrm{piecewise}(x < 10, 3\,x^2, 300)$$

```
> F := int( w(x), x=0..15);
```

$$F := 2500$$

```
> xbar := 1/F * int( x*w(x), x=0..15.);
```

$$xbar := 10.50000000$$

```
>
```