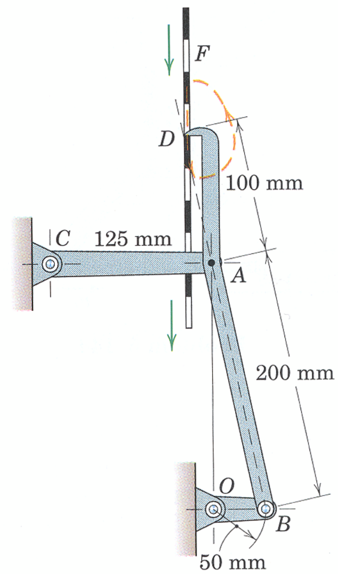
Review Problems - Final Exam



3. An intermittent-drive mechanism for a perforated tape F consists of the link DAB driven by the crank OB. The trace of the motion of the finger at D is shown by the dashed line. Determine the magnitude of the acceleration of D at the instant shown when both OB and CA are horizontal. Crank OB has a constant clockwise angular velocity of 120 rpm.
(taken from Dynamics by Meriam and Kraige, Fourth Edition)

ans: $a_D = 1997\ mm/s^2$

Strategy: pure kinematics!

Relate the acceleration at D to the acceleration at A as we work our way through member ABD:

$$\bar{a}_D = \bar{a}_A + \bar{a}_{D/A}$$
$$= \bar{a}_A + \bar{\alpha}_{ABD} \times \bar{r}_{D/A} - \omega^2_{ABD}\bar{r}_{D/A}$$

Expanding and equating components

$$\hat{i}: \quad a_{Dx} = a_{Ax} - \alpha_{ABD} r_{D/A_y} - \omega^2_{ABD} r_{D/A_x} \quad (1)$$
$$\hat{j}: \quad a_{Dy} = a_{Ay} + \alpha_{ABD} r_{D/A_x} - \omega^2_{ABD} r_{D/A_y} \quad (2)$$

unk	eqs
a_{Dx}	1
a_{Dy}	2
a_{Ax}	3
a_{Ay}	4
α_{ABD}	5
ω_{ABD}	6
a_{Bx}	7
a_{By}	8

Relate the acceleration at A to the acceleration at B

$$\bar{a}_A = \bar{a}_B + \bar{a}_{A/B}$$
$$= \bar{a}_B + \bar{\alpha}_{ABD} \times \bar{r}_{A/B} - \omega^2_{ABD}\bar{r}_{A/B}$$

Expanding and equating components

$$\hat{i}: \quad a_{Ax} = a_{Bx} - \alpha_{ABD} r_{A/B_y} - \omega^2_{ABD} r_{A/B_x} \quad (3)$$
$$\hat{j}: \quad a_{Ay} = a_{By} + \alpha_{ABD} r_{A/B_x} - \omega^2_{ABD} r_{A/B_y} \quad (4)$$

We can't get another two equations by relating the acceleration at D to the acceleration at B. Why?
We can't make any assumptions about the acceleration at A. Why?

Relate the acceleration at B to the acceleration at O since we know something about OB:

$$\bar{a}_B = \bar{a}_O + \bar{a}_{B/O}$$
$$= \bar{a}_O + \bar{\alpha}_{OB} \times \bar{r}_{B/O} - \omega^2_{OB}\bar{r}_{B/O}$$

Expanding and equating components knowing $\bar{\alpha}_{BO} = \bar{a}_O = \bar{0}$

$$\hat{i}: \quad a_{Bx} = -\omega^2_{OB} r_{B/O_x} \quad (5)$$
$$\hat{j}: \quad a_{By} = -\omega^2_{OB} r_{B/O_y} \quad (6)$$

Constraints and Geometry

Examining the velocity at points B and A, we see that member ABD is in translation at this instant. Thus the angular velocity is zero but we can draw no conclusions about the angular acceleration.

$$w_{ABD} = 0 \tag{7}$$

Examining the motion of point D (the dashed line), we see that has been in translation for more than an instant. We may therefore conclude that the acceleration of point D is in the y direction only.

$$a_{D\,x} = 0 \tag{8}$$

And for the position vectors:

$$\bar{r}_{D/A} = -25\hat{i} + 96.8\hat{j} \quad \Rightarrow \quad r_{D/A\,x} = -25mm,\ r_{D/A\,y} = 96.8mm$$
$$\bar{r}_{A/B} = -50\hat{i} + 193.6\hat{j} \quad \Rightarrow \quad r_{A/B\,x} = -50mm,\ r_{A/B\,y} = 193.6mm$$
$$\bar{r}_{B/O} = 50\hat{i} + 0\hat{j} \quad \Rightarrow \quad r_{B/O\,x} = 50mm,\ r_{B/O\,y} = 0mm$$