Rose-Hulman
Undergraduate Mathematics Conference

April 27, 2024

ROSE-HULMAN
INSTITUTE OF TECHNOLOGY

Department of Mathematics
Terre Haute, Indiana

Welcome

Welcome to the 39th Annual Rose-Hulman Undergraduate Mathematics Conference. This conference serves as an opportunity to highlight the research being done by undergraduate students which involves the use of mathematics. Students and faculty performing research across a wide spectrum are brought together by our shared interest in the mathematical sciences. Today is an opportunity to celebrate the accomplishments of those who are presenting, to encourage those for whom research is on the horizon, to broaden our knowledge among new and old friends, and to meet and converse others who also have a passion for all things mathematical. We have a great program this weekend.

Invited Speakers

Our speakers this year have some beautiful and interesting applications of mathematics to discuss. Dr. Robert Bosch, Professor of Mathematics at Oberlin College, will talk about applications of optimization to art. Dr. Lucy Muthoni, Scholar-in-Residence at Rose-Hulman, will discuss applications of probability to everyday decisions.

Contributed Papers

The focal point of the weekend is the contributed student talks. We have approximately 16 papers being presented by students this weekend. Regardless of your mathematical interests, we are certain you will find something that appeals to you. We are grateful to all those students who are willing to share their work with us during the conference. We are excited to host you this weekend, and we hope you enjoy the conference!

Rose-Hulman Undergraduate Math Journal

The Rose-Hulman Undergraduate Math Journal is devoted entirely to papers written by undergraduates on topics related to mathematics. Although the authors need not be undergraduates at the time of submission or publication, the work must have been completed before graduation. The journal is distributed freely in an electronic format from the journal's web site.

In order to maintain a high level of exposition, each paper is sponsored by a mathematician familiar with the student's work and each paper is refereed. The editor-in-chief makes the final decision for publication. The journal is sponsored by the Mathematics Department at Rose-Hulman Institute of Technology.

To each of our presenters, we hope you will consider publishing your research. Should you choose to consider the Rose-Hulman Undergraduate Math Journal, please visit the journal's website for more information

Phone: 812.877.8302
Email: mathjournal@rose-hulman.edu
Website: `scholar.rose-hulman.edu/rhumj`

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Schedule

Saturday

Donuts, Coffee, and Math

8:30 - 9:00am, New Academic Building Lobby
We will have donuts and coffee available as you start your day.

Introductory Remarks

9:00am Myers Hall M137
Official Welcome and start to the conference – Rick Stamper, Provost

Plenary Speaker

9:10 - 10:10am, Myers Hall M137 Dr. Robert Bosch, Single-Line Drawings via Mathematical Optimization

Morning Contributed Student Talks

10:20 - 11:35am, NAB J202 and J204
Due to the volume of student presentations, we are excited to have parallel sessions of student talks conveniently next door to each other.

NAB J202

Time	Presenter	Title
10:20 - 10:35	Tristan Rumsey	A Statistical and Machine Learning Look for the Influence of Common Soccer Metrics on Expected Goal Measures
10:40 - 10:55	Gracyn Swaggerty	An Exploration and Analysis of Fake Review
11:00 - 11:15	Alexa Renner	Braids and the Stranded Cellular Automata Model
11:20 - 11:35	Benjamin Glancy, Alexa Renner	Two new developments of central path in low dimensions

NAB J204

Time	Presenter	Title
10:20 - 10:35	Connor Lane	Cyclic Galois Extensions and Groups on Conic Sections
10:40 - 10:55	Benjamin Lyons	Patterns in the Cahn-Hilliard Equation with Long-Range Interactions
11:00 - 11:15	Taro Ikeda	Mathematical Isolation and Innovation: The Unique Development of 17th and 18th Century Japanese Mathematics
11:20 - 11:35	Jack DeLong	Elliptic Triangles Which Are Congruent To Their Polar Triangles

Lunch

11:45am - 12:45pm, New Academic Building Lobby
We will have a catered hot bar with a variety of options to meet most dietary restrictions and needs. Feel free to enjoy wherever you would like on campus.

Afternoon Contributed Student Talks

1:00 - 2:20pm, NAB J202 and J204
Due to the volume of student presentations, we are excited to have parallel sessions of student talks conveniently next door to each other.

NAB J202

Time	Presenter	Title
1:00 - 1:15	Roland Long	On Albert's Problems Involving Wired Wheels
1:20 - 1:35	Daniel Leverett	Challenges of Informational Asymmetry: Solving Games Marked by Both Incomplete and Imperfect Information
1:40 - 1:55	Jacob Olinger	Firefly Spark Classification Optimization
2:00 - 2:15	Owen Levens	Inversive Geometry on the Riemann Sphere

NAB J204

Time	Presenter	Title
1:00 - 1:15	Xiaoyu Liu	Scaling Limit of Asymptotically-Free Self-Interacting Random Walks to Brownian Motion Perturbed at Extrema
1:20 - 1:35	Samuel Lam	Predictability in College Sports: Comparing the Accuracies of Prediction Models for College Football and Basketball Games
1:40 - 1:55	Daryl Zazycki	Isomorphisms of Some Twisted Graded Calabi–Yau Algebras
2:00 - 2:15	Alexander Wilds	Splits and Substitutions: An All-in-One Calculus II Integral

Plenary Speaker

2:30 - 3:30pm, Myers Hall M137
Dr. Lucy Muthoni, Bringing 'x' to Life: The other Side of Mathematics

Closing Remarks

3:30 - 3:35pm, Myers Hall M137

Abstracts

Plenary Speakers

Single-Line Drawings via Mathematical Optimization
Robert Bosch, Oberlin College
9:10-10:10am, Myers Hall, M137

Optimization is concerned with optimal performance–finding the best way to complete a task. It has been put to good use in a great number of diverse disciplines: advertising, agriculture, biology, business, economics, engineering, manufacturing, medicine, telecommunications, and transportation (to name but a few). In this lecture, we will showcase its amazing utility by demonstrating its applicability in the area of visual art, which at first glance might seem to have no use for it whatsoever! Our focus will be on how various techniques in mathematical optimization can be used to design beautiful and intricate single-line drawings.

Bringing 'x' to Life: The other Side of Mathematics
Lucy Muthoni, Rose-Hulman Institute of Technology
2:30 PM - 3:30 PM in Myers Hall, M137

Mathematicians have often been accused of tasking students to find 'x'. This presentation aims to make it clear that we do not send students on a senseless task to look for our exes, but to bring sense to the data being presented to them for the sake of decision making.
The objective of this talk is threefold: first, to demystify the meaning and importance of random variables by explaining the basic meaning of input and output variables, then explain how they relate to each other through functions and how that translates to different applications, and to finally merge the two concepts into probability functions and showcase how these distributions contribute to decisions that affect our everyday lives.

Morning Undergraduate Speakers

A Statistical and Machine Learning Look for the Influence of Common Soccer Metrics on Expected Goal Measures
Tristan Rumsey, Butler University
10:20 - 10:35 AM in NAB J202

The advent of sports analytics has ignited a fervor across all sporting disciplines, particularly soccer, where clubs are sprinting to harness vast data reserves to elevate team performance, spearhead effective marketing endeavors, and bolster financial gains crucial for club expansion. Much like Billy Beane's transformative "Moneyball" approach, soccer clubs are in pursuit of innovative strategies to transcend financial limitations and achieve triumph. In soccer, where goals are scarce commodities, heightened offensive efficacy becomes imperative. Presently, one metric stands out as pivotal in gauging a team's goal-scoring success: expected goals (xG). This metric quantifies the likelihood of a given shot or opportunity culminating in a goal, making it a linchpin in a team's offensive strategy. Maximizing expected goals becomes paramount for teams aiming to capitalize on limited scoring opportunities during matches. Crucially, the first step in reshaping tactical approaches hinges on identifying the most influential variables in predicting expected goals. To this end, this study employs an array of machine learning methodologies, including Ridge, Lasso, Elastic Net, and Group Lasso models. The objective is to unveil the key predictor variables that significantly impact team (offensive) performance, often delineating the thin line between championship glory and defeat. With the aim of predicting xG, this research also incorporates modified bootstrap techniques to compute prediction intervals for the regularized machine learning models. By delving into the intricate fabric of soccer analytics, this study seeks to empower clubs with actionable insights, fostering a new era of strategy and competitive edge on the field.

Cyclic Galois Extensions and Groups on Conic Sections
Connor Lane, Rose-Hulman Insitute of Technology
10:20 - 10:35 AM in NAB J204

One of the main goals of Galois theory is understanding the Galois groups of all Galois extensions of a field. This is an extremely ambitious goal, so we typically reduce to certain subsets of extensions and fields with specified properties. One example of this reduction is Kummer theory, which considers cyclic extensions of fields that contain enough roots of unity. We construct an analogous theory that classifies cyclic Galois extensions of fields that contain enough torsion points of a conic section.

An Exploration and Analysis of Fake Reviews
Gracyn Swaggerty, Illinois State University
10:40 - 10:55 AM in NAB J202

Recently, online shopping has become the central platform for customers to purchase almost everything and hence reviews are now more important than ever for sellers as they can quickly sway customers away or toward a product. Reviews hold this influential weight because they are assumed to be authentic and coming from real customers with real experiences with the product. As a result, fake reviews and their detection has become a growing interest of various researchers.

For instance, Amazon Vine is widely used to verify reviewers. In this project we analyzed several reviews ranging from good, bad, and mixed, for various products. Our hypothesis testing aims to determine if there is a statistically significant difference between the expected frequency of some word- strings using the English Brown Corpus and the actual frequency found in the collected reviews.

Patterns in the Cahn-Hilliard Equation with Long-Range Interactions
Benjamin Lyons, Rose-Hulman Institute of Technology
10:40 - 10:55 AM in NAB J204

The Cahn-Hilliard equation is a partial differential equation that governs the behavior of a binary fluid system. In this work, we use a version of the Cahn-Hilliard equation that contains an additional term to account for the long-range interaction of the fluid molecules. We analyze the dynamic transitions and pattern formation of the model as we vary a system control parameter. One of the main goals of this work is to deduce necessary and sufficient conditions (on the system control parameter and fixed parameters) for the equilibria to form hexagonally packed cylinder patterns.

Braids and the Stranded Cellular Automata Model
Alexa Renner, Rose-Hulman Institute of Technology
11:00 - 11:15 AM in NAB J202

The Stranded Cellular Automata model is a model containing a grid of cells, and each cell can be in one of a finite number of states. Patterns can be represented on the Stranded Cellular Automata model with a turning rule, a crossing rule, and a set of initial conditions. In this talk, a criterion to determine which members of a braid group have a representation on the Stranded Cellular Automata Model will be presented. An algorithm to determine the compact SCA Algebra form of a braid group element will be presented as well, along with various properties of the algorithm.

Mathematical Isolation and Innovation: The Unique Development of 17th and 18th Century Japanese Mathematics
Taro Ikeda, Butler University
11:00 - 11:15 AM in NAB J204

In the 17th and 18th centuries, Western mathematics was revolutionized by figures like Newton, Leibniz, Euler, and Bernoulli. Japan's isolation fostered a parallel world of mathematical innovation. This presentation highlights the groundbreaking work of Seki Takakazu, a Japanese mathematician who independently approximated π and discovered concepts akin to the Bernoulli numbers and derivatives before their recognition in the West.

Two new developments of central path in low dimensions
Benjamin Glancy & Alexa Renner, Rose-Hulman Institute of Technology
11:20 - 11:35 AM in NAB J202

The central path is a set of solutions to a family of nonlinear programming problems associated

with a linear program. The central path is analytic for each description of a linear program, but the path can differ for different algebraic descriptions of the same linear problem. We extend typical analysis in two ways, those being 1) we establish a continuity result in a circle and 2) we show that central paths in a cylinder can be generated equivalently from a cube inscribed in the cylinder.

Elliptic Triangles Which Are Congruent To Their Polar Triangles
Jack DeLong, Aquinas College
11:20 - 11:35 AM in NAB J204

We perform the construction of a triangle congruent to its polar triangle and discuss properties.

Afternoon Undergraduate Speakers

On Albert's Problems Involving Wired Wheels
Roland Long, Northern Kentucky University
1:00 - 1:15 PM in NAB J202

Under contract with the National Security Agency, University of Chicago algebraist Abraham Adrian Albert authored several papers regarding the algebra of wired wheel cipher devices. The problems he considered concern the determination of a wired wheel's transformation from partial knowledge of successive alphabets. The earlier of these reports consider a machine of similar construction to Enigma, with other machine structures discussed in context of a simplified Enigma machine. Additionally, Albert explores "wheels without parallels," in which the difference between an element and its image cannot be repeated (i.e. a permutation sending 1 to 3 and 2 to 4 would be disallowed). In a wheel without parallels, the outputs of a reading station, analogous to inputting the same character for all stages of a machine, yield a permutation.

Scaling Limit of Asymptotically-Free Self-Interacting Random Walks to Brownian Motion Perturbed at Extrema
Xiaoyu Liu, Purdue University
1:00 - 1:15 PM in NAB J204

In this work, we show convergence of a family of one-dimensional self-interacting random walks to Brownian motion perturbed at extrema under the diffusive scaling. This completes the functional limit theorem developed by Kosygina, Mountford and Peterson for the asymptotically free case when $0 < p \leq 1/2$. The approach is to approximate the total drift experienced by the walker via analyzing directed edge local times, described by the branching-like processes. The analysis depends on the diffusive approximation of the branching-like processes obtained in the Ray-Knight type framework. This work is co-authored with Zhe Wang.

Challenges of Informational Asymmetry: Solving Games Marked by Both Incomplete and Imperfect Information
Daniel Leverett, Rose-Hulman Institute of Technology
1:20 - 1:35 PM in NAB J202

One of game theory's most impactful lines of development has been the expansion of its capacity to analyze asymmetric information distributions across players. Since the work of Nobel Prize-winner John Harsanyi, game theory has increasingly analyzed games marked by incomplete information, with some players lacking knowledge about other players' payoffs and preferences. Likewise, game theory has analyzed games with imperfect information, in which some players do not know all actions taken by other players. This project explores solution concepts for games marked by both incomplete and imperfect information. The analysis focuses on "social deduction" games, specifically, studying a well-known social deduction game. We model this as a Bayesian game, with solution concepts specifying payoffs for different player roles, identifying players' common prior beliefs about probability distributions of various roles, and updating Bayesian priors for each player's possible roles as a game unfolds. The solution concepts also incorporate effects of imperfect information. Finally, the project applies these solution concepts by implementing algorithms for efficiently solving social deduction games.

Predictability in College Sports: Comparing the Accuracies of Prediction Models for College Football and Basketball Games
Samuel Lam, University of Illinois
1:20 - 1:35 PM in NAB J204

How predictable are the outcomes of sports games? The predictability of a game tells us how closely matched the teams are, as well as how much the outcome depends on skill as opposed to luck.
To analyze this question, we consider three approaches to predicting game outcomes. The first is based on subjective rankings such as the weekly AP Top 25 polls; the higher ranked team is predicted to win. The second is based on mathematical models, such as Elo scores, for rating teams (originally designed to rate chess players); the team with the higher Elo score is predicted to win. The third approach is based on the betting market; the team favored by the betting market is predicted to win.
We apply these approaches to analyze and compare predictability of college football and college basketball games using historical game results data going back to 1950.

Firefly Spark Classification Optimization
Jacob Olinger, Rose-Hulman Institute of Technology
1:40 - 1:55 PM in NAB J202

Classification, a pivotal problem in computer science, often demands computationally expensive models. This paper delves into the optimization of firefly spark classification using the Apache Spark framework, leveraging the efficiency of stochastic algorithms and parallel processing. The Firefly algorithm, a swarm algorithm inspired by the movement of fireflies, is implemented within Spark, allowing concurrent evaluation of particles for optimal centroid determination in data classification. In the pursuit of more efficient model training, stochastic algorithms have gained promi-

nence due to their population-based searchability. Apache Spark, with its parallel processing capabilities, allows the Firefly algorithm to be parallelized to enable faster processing. In particular, this study focuses on implementing the Firefly algorithm within the Spark framework for classification tasks, employing the benefits of parallelism in large-scale data processing. The Firefly algorithm, originating from bio-inspired algorithms, involves the random and also guided movement of particles toward the brightest or fittest direction over defined iterations. The integration of Spark enables independent processing of each particle by worker nodes, facilitating concurrent evaluation during the algorithm iterations. The study compares the algorithm's performance on two datasets: EEG-eye state and poker. Linear speed-up values are observed with increasing node numbers, particularly in larger datasets. The EEG dataset, with binary targets, exhibits less proportional speed-up. In contrast, the poker dataset, with multi-classification targets, demonstrates a more proportional speed-up, emphasizing the impact of the concurrency potential on the Firefly algorithm. Results underscore the efficiency gains achievable through the Spark-enabled, nature-inspired Firefly algorithm. The linear speed-up values with larger datasets highlight the advantages of parallel processing in Spark for big data applications. Future work will explore concurrent optimizations, refinement of the fitness functions for improved clustering accuracy, and extended testing to a wider range of datasets. This experimental implementation showcases the potential of the Spark-enabled Firefly algorithm for efficient large-scale data classification.

Isomorphisms of Some Twisted Graded Calabi–Yau Algebras
Daryl Zazycki, Miami University
1:40 - 1:55 PM in NAB J204

We solve the isomorphism problem for a certain family of twisted graded Calab-Yau algebras. These algebras appear as quotients of certain path algebras on a quiver, and can be realized as twists of preprojective algebras of type A.

Inversive Geometry on the Riemann Sphere
Owen Levens, DePaul
2:00 - 2:15 PM in NAB J202

Join me on an exciting journey through transformations of the complex plane joined with the point at infinity–otherwise known as the Riemann Sphere. We'll start by placing a penny in the complex plane and sending everything outside inside and everything inside outside–the center goes to the point at infinity, and the point at infinite goes to the center. We'll observe this mapping to be continuous, differential, conformal, and analytic–preserves angles, intersections, and tangencies. We will explore which geometric configurations are invariant, how they are transformed (which circles are sent to circles and lines), and unpublished results in my research in the composition of these maps.

Splits and Substitutions: An All-in-One Calculus II Integral
Alexander Wilds, Indiana State University
2:00 - 2:15 PM in NAB J204

This indefinite integral arising from the use of the Integral Test turns out to include many of the

most common integration techniques, and is - perhaps surprisingly - completely solvable with no more than a basic Calculus II course. The techniques used to solve this integral include partial fraction decomposition, u-substitution, and trigonometric substitutions. The solution arrived at by these methods is also compared to results from online software such as WolframAlpha.