# Rose-Hulman Undergraduate Mathematics Conference 

April 19-20, 2019


Rose-Hulman Institute of Technology
Department of Mathematics
Terre Haute, Indiana

## Welcome

Welcome to the $36^{\text {th }}$ Annual Rose-Hulman Undergraduate Mathematics Conference. This conference serves as an opportunity to highlight the research being done by undergraduate students which involve the use of mathematics and statistics. Students and faculty performing research across a wide spectrum are brought together by our shared interest in the mathematical sciences. This weekend is an opportunity to celebrate the accomplishments of those who are presenting, to encourage those for whom research is on the horizon, to broaden our knowledge amongst new and old friends, and to socialize with others who also have a passion for mathematics and statistics. We have a great program this weekend.

Invited Speakers: Mathematics have several applications, but many find joy in mathematics because of its beauty. This beauty is highlighted by the invited speakers at this year's conference. Dr. Susan Goldstine, Professor of Mathematics at St. Mary's College of Maryland, will challenge us to find the beauty in the impossible. Dr. Mark Bollman, Chair of the Department of Mathematics at Albion College, will help us appreciate complexity in games of chance.

Short Courses: We are pleased to offer three short courses to our registrants. These two-hour courses will present foundational topics in mathematics and statistics. Continuing to find beauty in mathematics, Dr. Josh Holden (Rose-Hulman Institue of Technology) will offer a course on the mathematics of weaving. Of special interest to faculty this year, Dr. Tim All (Rose-Hulman Institute of Technology, and local TeX guru) will offer a course on advanced editing in LaTeX. Dr. Mark Daniel Ward (Purdue University) will provide an introduction to data science for those seeking an applied course.

Contributed Papers: The focal point of the weekend is the contributed student talks. We have 23 papers being presented by students this weekend on topics ranging from the mathematics of cloaking to a simulation to better understand the domestication of wolves. Regardless of your specialty, we are certain you will find something that interests you. We are grateful to all those students who are willing to share their work with us during the conference.

We are excited to host you this weekend, and we hope you enjoy the conference!

## Acknowledgments

This conference would not have been possible without the dedication and service of many. We are grateful to all those who have helped this conference come together. In particular, we would like to thank the following individuals:

| Dr. Anne Houtman | Provost and Vice President of Aca- <br> demic Affairs |
| :--- | :--- |
| Mariana Lane | Administrative Assistance |
| Michelle Prather | Administrative Assistance |

To all those we neglected to mention above, we truly are appreciative of all the work you put into the conference.

And to all those participating in the conference, thank you for attending; you are what the conference is all about.

## Rose-Hulman Undergraduate Math Journal

The Rose-Hulman Undergraduate Math Journal is devoted entirely to papers written by undergraduates on topics related to mathematics. Although the authors need not be undergraduates at the time of submission or publication, the work must have been completed before graduation. The journal is distributed freely in an electronic format (PDF) from the journal's web site.

In order to maintain a high level of exposition, each paper is sponsored by a mathematician familiar with the student's work and each paper is refereed. The editor-in-chief makes the final decision for publication. The journal is sponsored by the Mathematics Department at Rose-Hulman Institute of Technology.

To each of our presenters, we hope you will consider publishing your research. Should you choose to consider the Rose-Hulman Undergraduate Math Journal, please visit the journal's website or contact the following individuals:

Dr. Tim All<br>Dr. Joe Eichholz<br>Dr. Josh Holden

Dr. Tom Langley Dr. John McSweeney
Department of Mathematics
Rose-Hulman Institute of Technology

Phone: 812.877.8391
Email: mathjournal@rose-hulman.edu
Email: scholar.rose-hulman.edu/rhumj

## Terre Haute

During your stay in Terre Haute, we anticipate you will spend most of your time on the Rose-Hulman campus. Should you decide to explore Terre Haute, for those unfamiliar, there are four key roads that form a box: US 40 (Wabash Avenue) on the North, Interstate 70 on the South, US 41 (Third Street) on the West, and State Road 46 on the East.

## Accommodations

The conference hotel is the Quality Inn of Terre Haute, located at 555 South Third Street (812.235.3333). As you exit the Rose-Hulman campus, turn right onto Wabash Avenue. Proceed through downtown Terre Haute until you reach the court house; turn left on Third Street. The hotel will be on the right after a few blocks. Please remember, the conference hotel offers a hot breakfast each morning!

We thank you for choosing to stay at the conference hotel. Your patronage keeps housing costs lower, allowing us to subsidize the cost for many students attending the conference.

## Restuarants

Terre Haute is home to many large chains. On State Road 46, just off Interstate 70, there are a few fast food options and a Mexican restaurant located near the Walmart complex. Downtown Terre Haute offers some local restaurants; local favorites include Mogger's and Saratoga. Third Street, near Interstate 70 and the Honeycreek Mall offers several options including fast food and larger chains.

While the conference is providing dinner on Friday evening and lunch on Saturday afternoon, conference attendees are responsible for lunch on Friday and breakfast Saturday morning. The conference hotel will provide breakfast on Saturday morning for those staying at the Quality Inn. If you are not staying at the conference hotel, there are several breakfast options (Bob Evans, Cracker Barrel, Denny's, McDonald's, etc.) on Third Street. The conference will provide light refreshments on Saturday morning.

## Floor Plans

All conference activities will take place in the Mussallem Student Union. Registration, short courses, and contributed student talks will take place on the upper level of the union, while dinner and plenary sessions will take place on the lower level.




## Program Summary

| Friday, April 19 |  |  |
| :---: | :---: | :---: |
| 12:00pm - $3: 00 \mathrm{pm}$ | Conference Registration and Onsite Check-In | Mussallem Union (Upper Level) |
| 1:30pm - 3:30pm | Short Courses | Mussallem Union (Upper Level) |
|  | Introduction to Data Science <br> Dr. Mark Daniel Ward | B-111 |
|  | Mathematics of Weaving <br> Dr. Joshua Holden | MU-262 |
|  | Upping your LaTeX Game: Class files, scripting, and graphics Dr. Timothy All | KIC-158 |
| 3:50pm - 5:25pm | Contributed Papers | Mussallem Union (Upper Level) |
| 3:50pm - 4:10pm | Establishing the Method of Moments Estimation for Subdiffusion Processes with $\beta$-Stable Waiting Times Phillip Kerger | MU-233E |
|  | Solving the Dirac System with the Unified Transform Method Casey Garner | MU-233W |
| 4:15pm - 4:35pm | Estimating Traffic Intensity with Semantic Segmentation Logan Bradley-Trietsch | MU-233E |
|  | Information Fusion and Boolean Algebra Kathryn Schantz | MU-233W |
|  | Proof of a Construction of Odd Order Magic Cubes Joshua Arroyo | MU-260 |
| 4:40pm - 5:00pm | Wolf Domestication: An Agent-Based Simulation Ryan Kulwicki | MU-233E |
|  | Well-Poised Hypersurfaces Angela Vichitbandha | MU-233W |
|  | Ineffective Sets and the Region Crossing Change Operation Rachel Morris | MU-260 |
| 5:05pm - 5:25pm | Diversity of forest structure across the United States Jessica Gilbert | MU-233E |
|  | Diophantine Equations and Quadratic Integer Rings Rose Schweizer | MU-233W |
| 5:30pm - 6:30pm | Dinner | Mussallem Union (Lake Room, Lower Level) |
| 6:45pm - 8:00pm | Plenary Session | Mussallem Union (Lake Room, Lower Level) |

When Mathematics Says No: The Aesthetics of Impossibiliy
Dr. Susan Goldstine

## Saturday, April 20

| 8:30am - 9:00am | Conference Registration and Onsite Check-In | Mussallem Union (Upper Level) |
| :---: | :---: | :---: |
| 9:00am-10:00am | Plenary Session | Mussallem Union (Lake Room, Lower Level) |
|  | The Mathematics of Blackjack Dr. Mark Bollman |  |
| 10:15am-11:50am | Contributed Papers | Mussallem Union (Upper Level) |
| 10:15am - 10:35am | Impact of the Economic Recession on Measures of Health Luke Francisco | MU-233E |
|  | Power Graphs of Finite Groups Allen Williams | MU-233W |
|  | Almost Beatty Partitions and Optimal Scheduling Problems Xiaomin Li | MU-260 |
| 10:40am - 11:00am | A computational approach to the structure of subtraction games Kali Lacy | MU-233E |
|  | Wallace-Simson and Monsky Theorems in non-Euclidean Geometries <br> Kelsey Hall | MU-233W |
|  | Tilings of a Domain of the Triangular Lattice Jessica Bai, Leonardo Rodriguez, Congwei Yang | MU-260 |
| 11:05am-11:25am | Generating Interest in Generating Functions (And more word play) Kevin LaMaster | MU-233E |
|  | Spectral Dynamics for Second Order Differential Equations with Singular Weight <br> Son Nguyen | MU-233W |
|  | Cloaking, Wave Equation, and Damping Models Muqing Zheng | MU-260 |
| 11:30am - 11:50am | Big Bad Matrices: A Constructive Approach Garrett Mulcahy | MU-233E |
|  | A Symmetric Random Walk on a Reinforced Cube Madelyn Nolting | MU-233W |
|  | Partitioning Cellular Automaton and Hexagon Lattice Gases Jiawen Wang | MU-260 |
| 12:00pm - 12:50pm | Lunch \& Panel Discussion | Mussallem Union (Lake Room, Lower Level) |

## Program | Short Courses

Introduction to Data Science

Dr. Mark Daniel Ward
Associate Professor of Statistics, Purdue University
Time: Friday, 1:30pm - 3:00pm
Location: B-111 (GM Room)
We will have a hands-on overview of some of the tools that data scientists use for working with data, including large data sets. The workshop topics can be slightly flexible and open to discussion, depending on the interests of the participants. At a minimum, we will introduce students to R and RStudio, data visualization, and perhaps some tools for scraping and parsing XML directly from the web and processing the scraped data in $\mathrm{R} .{ }_{\mathrm{j}} \mathrm{U}+00 \mathrm{~A} 0$;All participants are encouraged to bring a laptop... and to be excited to learn about some of the introductory nuts and bolts of data science. No computational background is needed for this workshop.

## Mathematics of Weaving

## Dr. Joshua Holden

Professor of Mathematics, Rose-Hulman Institute of Technology
Time: Friday, 1:30pm - 3:30pm
Location: MU-262
Weaving is one of the most mathematical of all art forms, and it's got something for every mathematician - geometry, topology, combinatorics, number theory, algebra, and even a little bit of differential equations. We'll take a hands-on tour of lots of different ways to explore mathematics with weaving. Be prepared to learn and experiment with various techniques of weaving paper strips. We will also demonstrate weaving with yarn on a loom and there may be opportunities for you to try that out, too. No previous knowledge of weaving or any particular area of mathematics is required. Participants are encouraged to bring your creativity! All materials will be provided.

Upping your LaTeX Game: Class files, scripting, and graphics Dr. Timothy All
Assistant Professor of Mathematics, Rose-Hulman Institute of Technology
Time: Friday, 1:30pm - 3:30pm
Location: KIC-158
$\mathrm{LAT}_{\mathrm{E}} \mathrm{Xis}$ the de facto standard when it comes to typesetting mathematics and scientific documents. In this workshop, we aim to give an overview of some of the more advanced features available in ${ }^{\mathrm{LA}} \mathrm{T}_{\mathrm{E}} \mathrm{X}$. Specifically, we will explore the following:

- the creation of custom packages and document classes to avoid those mile-long preambles,
- other TeX engines with specific emphasis on LuaLatex which includes Lua as an embedded scripting language. This allows for a host of useful applications (e.g., creating custom assignment classes that auto-populate a table of scores or an answer key, or that randomly generate similar problems upon compiling),
- the various tools and add-ons available for creating high-quality 2D and 3D vector graphics in $\mathrm{EAT}_{\mathrm{E}} \mathrm{X}$.
Templates and/or minimal working examples will be provided for those who wish use or otherwise tootle around with those demos used in the workshop.


## Program | Plenary Sessions

When Mathematics Says No: The Aesthetics of Impossibiliy<br>Dr. Susan Goldstine<br>Professor of Mathematics, St. Mary's College of Maryland<br>Time: Friday, 6:45pm - 8:00pm<br>Location: Lake Room, Mussallem Union

Sometimes, when we pose questions of mathematics, its answers are strikingly contrary. Why can't we trisect an angle with the same tools we use to bisect an angle? It's not possible. Why haven't we found the quintic formula? It doesn't exist. Can we at least prove that arithmetic is logically consistent? Nope!

We can view these results as intransigent obstacles to human knowledge, or we can accept them as fascinating illustrations of the boundaries of different mathematical techniques. In this talk, we will explore analogous results for techniques in the fiber arts. For each form of knitting, crochet, embroidery, and so forth, there is a set of limitations on what types of designs they can produce. Sometimes, these limits are broad enough that the the art form can encompass every mathematical possibility. Other times, the craft imposes intriguing restrictions on what patterns we can produce, and we will make the case that these restrictions have their own intrinsic beauty.

About Susan: Susan Goldstine received her A.B. in Mathematics and French from Amherst College in 1993 and her Ph.D. in Mathematics from Harvard University in 1998. Her joint and individual artworks have appeared in Math Horizons, the Journal of Mathematics and the Arts, the proceedings of the international Bridges Conference, and various mathematical art exhibits in the US and around the world. Together with computer scientist and artist Dr. Ellie Baker, she is coauthor of the 2014 book Crafting Conundrums: Puzzles and Patterns for the Bead Crochet Artist, which collects their extensive research on mathematical bead crochet. Dr. Goldstine is Professor of Mathematics at St. Mary's College of Maryland, where she has been on the faculty since 2004, and an Associate Editor for the Journal of Mathematics and the Arts. Her guiding principle is that a professor's office can never have too many toys.

## The Mathematics of Blackjack

## Dr. Mark Bollman

Chair of the Department of Mathematics and Computer Science, Albion College
Time: Saturday, 9:00am - 10:00am
Location: Blackjack, or 21, is among the most popular casino table games. Since, unlike most other games of chance, successive hands of blackjack are not independent, the mathematics behind blackjack is at once more complicated and more interesting than for games like craps or roulette, and there can be times during play when the gambler has an edge over the casino. This talk will briefly review the rules of the game and then describe some of the calculations-both theoretical and experimental-that led to blackjack basic strategy and the advantages derived from card counting.

About Mark: Mark Bollman is Professor of Mathematics and Chair of the Department of Mathematics and Computer Science at Albion College in Albion, MI. He earned a B.A. in mathematics and integrated science from Northwestern University, an M.A. in mathematics from the University of Michigan, and a Ph.D. in mathematics, with a concentration in college teaching of mathematics, from Central Michigan University. Mark's mathematical interests include number theory, probability, and geometry. Recently, he has been working in applied probability, specifically, the application of mathematics to games of chance, and has published two books, Basic Gambling Mathematics: The Numbers Behind The Neon and Mathematics of Keno and Lotteries, both with CRC Press. He has traveled with his gambling math students to casinos in Michigan and Nevada to visit the origins of their subject and compare theory and practice. Mark has taught 112 different college courses in his career, including classes in computer science, English composition, physics, and chemistry as well as all levels of mathematics.

## Program | Contributed Papers

Abstracts of contributed papers are listed below.

## 3:50 PM to 4:10 PM

Establishing the Method of Moments Estimation for Subdiffusion Processes with $\beta$ Stable Waiting Times<br>Phillip Kerger, Fordham University<br>Location: MU-233E

In this paper, we establish an method-of-moments-type estimation method for the parameter involved in stochastic processes time-changed by a stable subordinator. We use only the number of constant periods observed in the process to define the estimator. We provide approximations for the variance and the bias of the estimator, and compare its performance to the CDF curve-fitting method in simulations. After adjusting for bias, the resulting estimator is quick to compute and especially well-suited for situations in which multiple observances of the same process are available. We continue with an application of this method to market price data for low-volume stocks, estimating the parameter with the method-of-moments-type estimator and comparing resulting estimates to those of the CDF curve-fitting method. The estimator produces similar results with significantly less computational effort, so it is well-suited for situations in which many estimates need to be made or in which estimates are made over large amounts of data.

## Solving the Dirac System with the Unified Transform Method

Casey Garner, Rose-Hulman Institute of Technology
Location: MU-233W
The Unified Transform Method is a recently developed method for solving boundary value problems. This method subsumes classical methods such as separation of variables and Fourier series, and has several beneficial properties. In this talk, we will discuss how we utilized the Unified Transform Method to solve the Dirac system of partial differential equations, a system of equations developed by Dirac to model quantum mechanical particles. In addition, we will highlight the key concepts behind the method and explain its utility.

## 4:15 PM to 4:35 PM

## Estimating Traffic Intensity with Semantic Segmentation

## Logan Bradley-Trietsch, Purdue University

Location: MU-233E
The computer vision task semantic segmentation, which involves assigning a label to each pixel in an image, has many applications to vital topics such as medical imaging or autonomous driving. This project uses the semantic segmentation deep learning architecture Deeplabv3+ to make inferences about the intensity of vehicular and pedestrian traffic. This unique approach to traffic estimation has advantages over traditional methods like object detection in its ability to more accurately account for both large and small vehicles that each affect traffic differently. This project and its methods are one step toward the "smart city", i.e., a city that has a traffic system based on artificial intelligence that can adapt to traffic in real-time with the end goal of mitigating gridlock traffic in large cities.

## Information Fusion and Boolean Algebra

Kathryn Schantz, University of Kentucky
Location: MU-233W
In this talk, I will introduce the concepts of a detection system and information fusion, then will discuss using ROC (receiver operating characteristic) curves to quantify the accuracy of detection systems. I will discuss Boolean rules for fusing different detection systems and the mathematical structures that result from this fusion. I will justify results that at first seem surprising; in this context, Boolean rules do not always work as expected.


#### Abstract

Proof of a Construction of Odd Order Magic Cubes Joshua Arroyo, Rose-Hulman Institute of Technology Location: MU-260 A magic cube of order n is an $\mathrm{n} \times \mathrm{n} \mathrm{x} \mathrm{n}$ array filled with $n^{3}$ distinct positive integers $1,2, \ldots, n^{3}$ such that the n integers in each row, column, pillar and space diagonal all sum up to the magic sum. A magic cube of any odd order can be constructed through an algorithm that places elements in order along the 2-dimensional backward diagonals of the cube. We prove that our construction gives a magic cube by first deconstructing the magic cube into 4 cubes each containing the entries $0,1, \ldots,(n-1)$. Then we show that the sum of the entries of the rows, columns, pillars, and space diagonals are all the same. Time permitting, we show that the algorithm extends to create magic hypercubes of odd dimensions.


## 4:40 PM to 5:00 PM

## Wolf Domestication: An Agent-Based Simulation <br> Ryan Kulwicki, Valparaiso University <br> Location: MU-233E

Wolves are among the earliest known animals to be domesticated. However, the mechanism by which gray wolves were domesticated into dogs is still unknown. The prevailing domestication hypothesis is that humans selectively bred the gray wolves that were more docile. However, there is a more recent hypothesis which states that wolves which were less hostile towards humans would essentially domesticate themselves by naturally selecting for such wolves because of the availability of food near human settlements. Simulating these conditions could help prove the possibility of domestication via natural selection. Previously published mathematical models are based on systems of differential equations, and these models have critical simplifications such as homogeneous randomly mixed populations and infinite population sizes. Therefore, we created an agent-based simulation which has single trait evolution, user-defined and literature-based parameters, and sexual reproduction.

## Well-Poised Hypersurfaces

Angela Vichitbandha, University of Kentucky
Location: MU-233W
I will introduce some geometric and combinatorial properties of hypersurfaces. In particular, I will be discussing a class of hypersurfaces called "well-poised" which exhibit strong irreducibility properties. This property is revealed through the combinatorics of the Newton polytope of the hypersurface.

## Ineffective Sets and the Region Crossing Change Operation

Rachel Morris, University of Richmond
Location: MU-260
Region crossing change ( RCC ) is an operation performed on a selected region of a link diagram that reverses all crossings incident to that region. A set of regions are ineffective if, after RCC moves are performed on each region in the set, the resulting link diagram has the same crossing information as the original one. Ineffective sets are key to understanding how many RCCs it takes to transform one diagram the other. In this talk, we characterize the ineffective sets for all knots and reduced link diagrams. These are determined by checkerboard shading and a variant of checkerboard shading called tri-coloring. As an application, we will determine the maximum number of RCC moves needed to transform one knot diagram to another diagram with the same underlying projection.

## 5:05 PM to 5:25 PM

## Diversity of forest structure across the United States <br> Jessica Gilbert, Purdue University <br> Location: MU-233E

Forest ecosystems are constantly changing due to factors such as climate change, invasive species, and management. Traditionally, changes to forest ecosystems are described by shifts in species composition, successional stage, or phylogenetic diversity. Recent advancements in remote sensing, particularly LiDAR, have revealed the importance of stand structure (the arrangement of trunks, branches, and leaves in 3-dimensional space) on forest dynamics. However, the limited availability of LiDAR data at a continental scale prevents the analysis of forest structure across the United States. In this exploratory analysis, we compiled data from the U.S. Forest Inventory and Analysis (FIA) Program from over 120,000 plots. We computed a suite of metrics related to the distribution of tree size classes, species diversity, and phylogenetic diversity in FIA plots and mapped the resulting metrics across the U.S. In addition, we subsetted our data by ecoregion and assessed the relationships between these metrics. We found a positive relationship between forest structural diversity and both species and phylogenetic diversity at low levels of structural diversity, but the relationships tend to flatten out or become negative in areas with higher levels of structural diversity. These relationships also varied in strength between ecoregions which may be related to the regional variability in the species pool. Moving forward, we plan to explore the relationship between forest structure and ecosystem functions, which will help broaden our understanding of forest structural dynamics in a changing global environment.

## Diophantine Equations and Quadratic Integer Rings

Rose Schweizer, Hillsdale College
Location: MU-233W
This talk will explore how quadratic integer rings, such as $Z_{i}$, can be used to solve Diophantine equations. While classical questions such as which numbers can be written as a sum of squares are difficult to solve using number theoretic methods, using more abstract methods can simplify the solutions. These solutions depend on the properties of the particular quadratic integer ring, including the structure of the ideals within the ring. Some of these properties can be seen graphically as well as algebraically. In particular, the unique factorization of an element into primes is reflected by the tiling of the plane with parallelograms generated by the ideals of the ring.

## 10:15 AM to 10:35 AM

Impact of the Economic Recession on Measures of Health
Luke Francisco, Purdue University
Location: MU-233E
This presentation analyzes the impact of the Great Recession (2007-2009) on age-related declines in health. Previous research has yielded mixed findings regarding whether economic downturns are associated with periods of deteriorating or improving health. This research attempts to contribute a study making extensive use of statistical matching methods to this collection of results. Using data from three waves of the Survey of Mid-Life Development in the United States (MIDUS), a national study of health and well-being in adults, this study hypothesizes that those who lived through a stressful period such as the Great Recession will experience significantly greater declines in physical and mental health than those who lived through a period prior to the recession. The study combines subjects into a target group who were surveyed on two separate occasions spanning a period containing the onset of the recession. Each of these subjects is paired with a subject from the comparison group who was surveyed on two separate occasions prior to the recession. Pairings are based on the results of a nearest neighbor matching algorithm performed on a series of explanatory variables. Changes in self-reported functional limitations, depression, and self-rated health are used as response variables to determine differences in health outcomes between the matched pairs. The study results indicate that contrary to the hypothesis, the Great Recession was associated with improved health outcomes. This presentation is based on a study supported by the National Science Foundation under Grant No. 1246818.

Power Graphs of Finite Groups
Allen Williams, University of Toledo
Location: MU-233W
The Directed Power Graph of a group is a graph whose vertex set is the elements of the group, with an edge from x to y if y is a power of x . The Power Graph of a group can be obtained from the directed power graph by disorienting its edges. This work discusses properties of cliques, cycles, paths, and coloring in power graphs of finite groups. A construction of the longest directed path in power graphs of cyclic groups is given, along with some results on distance in power graphs. We discuss the cyclic subgroup graph of a group and show that it shares a remarkable number of properties with the power graph, including domination number and independence number.

## Almost Beatty Partitions and Optimal Scheduling Problems

Xiaomin Li, University of Illinois
Location: MU-260
A Beatty sequence is a sequence of the form [a*n], where a is an irrational number and the bracket denotes the floor function. A remarkable result, called Beatty's Theorem, says that if a and b are irrational numbers such that $1 / a+1 / b=1$, then the associated Beatty sequences "partition" the natural numbers. That is, every natural number belongs to exactly one of these two sequences. It is known that Beatty's Theorem does not extend directly to partitions into three or more sets, and finding appropriate analogs of Beatty's Theorem for such partitions is an interesting and wide open problem, which has applications to optimal scheduling problems. One example of such a problem is the "Chairman Assignment Problem" due to Robert Tijdeman: Suppose that k states with k positive weights (which sum up to 1 ) form a union. Each year, $\mathrm{i} U+00 \mathrm{~A} 0 \dot{j}$, a state is selected from which the Chair of the union is to be chosen. The problem asks for the optimal assignment of these states such that, for each $n$ and each state, the actual count of Chairs from this state in the first $n$ years is closest to the expected count, given by $n$ times the weight of the state. $\mathrm{U}+00 \mathrm{~A} 0$ 。 Tijdeman gave an algorithm to generate the optimal assignment. $\mathrm{j} \mathrm{U}+00 \mathrm{~A} 0 ;$ In the case $\mathrm{k}=2$ the resulting partition of the natural numbers turns out to be a partition into non-homogeneous Beatty sequences. In our research we consider the case $\mathrm{k}=3$. We construct partitions of the natural numbers into three sequences that are in some sense closest to actual Beatty sequences, and we apply these results to optimal scheduling problems, such as the Chairman Assignment Problem.

## 10:40 AM to 11:00 AM

A computational approach to the structure of subtraction games
Kali Lacy, Purdue University
Location: MU-233E
We present a computational methodology for the structure of subtraction games. One of the oldest problems in combinatorial game theory is to characterize the structure of subtraction games. Although the structure can be analyzed recursively, at present, a methodology for explicitly characterizing the structure of a subtraction game is not (yet) known. In the last two years, our team characterized the (eventual) period lengths of the Sprague-Grundy values of subtraction games with 3 parameters. Recently, however, we greatly generalized these results, to fully characterize the complete sequences of SG-values, including both the periodic and the pre-periodic portions of the sequences. We have analyzed 72 PB of data about this problem, to verify this computational approach to the analysis of these games.

Wallace-Simson and Monsky Theorems in non-Euclidean Geometries
Kelsey Hall, Aquinas College
Location: MU-233W
We will prove a Wallace-Simson-type theorem in elliptic geometry and give some counter-examples to Monsky's theorem in hyperbolic geometry.

## Tilings of a Domain of the Triangular Lattice

Jessica Bai, Leonardo Rodriguez, Congwei Yang, University of Illinois Location: MU-260
We consider the problem of tiling a rectangular domain of the triangular lattice by means of the six elementary tiles obtained by cutting a regular hexagon in half through opposite vertices such that each tile is composed of three adjacent triangles. For each rectangular domain of size $3 n$, there are a number of ways to uniquely tile the domain completely. The number of unique tilings changes depending on the size of the domain. To determine the number of unique tilings that can be made, we used transformation matrices, which represent different qualities of each tile. To visualize the many possibilities, we made use of a search tree program that generates all unique tilings of any dimension by trying every possible combination of ordering in which the tilings are placed. This program works for any domain but, due to the way it works, is very inefficient.

## 11:05 AM to 11:25 AM

## Generating Interest in Generating Functions (And more word play) <br> Kevin LaMaster, Purdue University <br> Location: MU-233E <br> Generating functions are an extremely valuable tool to solve and generalize probability and counting problems. We will focus on their applications to extracting information about words formed from a geometrically distributed alphabet. Using generating functions we can take this complicated probability problem and solve it in a very generalizable way. We will also demonstrate other uses including the counting and asymptotic analysis of Wolf partitions

Spectral Dynamics for Second Order Differential Equations with Singular Weight Son Nguyen, DePauw University
Location: MU-233W
In this talk, we investigate the eigenvalues and corresponding eigenfunctions of second order ordinary differential equations (ODE) with singular weight. We employ general techniques in ODE and Mathematica programming to visualize the behavior of the dynamics of eigenvalues and eigenfunctions.

## Cloaking, Wave Equation, and Damping Models

Muqing Zheng, Rose-Hulman Institute of Technology
Location: MU-260
In this paper, we provide a method for "cloaking" the outside world from an observer inside a fixed region. It is assumed the observer uses emitted and reflected waves-like radar-to image the exterior of the region. Our approach uses the idea of surrounding the region containing the observer with a special material, an absorbing buffer layer that does not reflect waves. As a result, the observer thinks the waves have traveled away to "infinity," and is unaware of the even the presence of the buffer. We discuss two for the damping model, and their properties. We then optimize the cloaking effect based on the balance between the decay rate of the wave amplitude and the thickness of the surrounding materials. From there, we investigate a much more efficient cloaking scheme with an extended property of the surrounding materials and the possibility of perfect cloaking. We also analyze an existing cloaking scheme, the so-called "perfectly matched layer" (PML). Finally, we present numerical examples showing the efficiency of cloaking for the two different choices, and compare them to the result of the PML scheme.

## 11:30 AM to 11:50 AM

## Big Bad Matrices: A Constructive Approach

Garrett Mulcahy, Purdue University
Location: MU-233E
John von Neumann "Approximative Properties of Matrices of High Finite Order" (1941) explores the asymptotic behavior of matrices as dimension increases but remains finite. Essentially, von Neumann ventured to explore the neglected middle ground between finite and infinite dimensional analysis. The major result of his paper is a proof of the existence of "big, bad matrices" that is, matrices of large dimension that possess "bad" qualities. von Nuemann's proof was nonconstructive, making use of what he called a "volumetric" argument. We utilize computational techniques in a quest to find a construction of these matrices; discovering what the matrices look like will potentially have applications to data science and the theory of random matrices.

## A Symmetric Random Walk on a Reinforced Cube Madelyn Nolting, IUPUI <br> Location: MU-233W

Construct a wire network of a cube. Reinforce it by augmenting all twelve face diagonals. Consider a symmetric random walk on this reinforced cube: In each step, a particle moves from one vertex to any one of the six adjacent vertices (except the one diametrically opposite to it) with equal probability. We answer three questions: (1) What is the distribution of the number of steps taken until the particle returns to its starting vertex? (2) What are the mean and the variance of the cover time (time to visit all vertices)? (3) Among all vertices, which vertex will be visited the last?

## Partitioning Cellular Automaton and Hexagon Lattice Gases <br> Jiawen Wang, Rose-Hulman Institute of Technology

Location: MU-260
A cellular automaton is a type of mathematical system that models the behavior of a group of cells with discrete values in progressing time steps. The often complicated and systematic behaviors of cellular automaton are studied in computer science, mathematics, biology, and other science related fields. This thesis focuses on discussing and analyzing the patterns that appear in some elementary cellular automaton and hexagon lattice gases. A python program was implemented to help visualize and analyze the behavior of the hexagon lattice gases model.

## Conference Attendees

| Ty Adams | Rose-Hulman Institute of Technology |
| :---: | :---: |
| Abdullrahaman Alasiri | Rose-Hulman Institute of Technology |
| Zeinab Aly | Purdue University |
| Shuto Araki | DePauw University |
| Joshua Arroyo | Rose-Hulman Institute of Technology |
| Jessica Bai | University of Illinois |
| Suman Balasubramanian | DePauw University |
| Rachel Benington | Rose-Hulman Institute of Technology |
| Logan Bradley-Trietsch | Purdue University |
| Joshua Bressman | Rose-Hulman Institute of Technology |
| Katherine Brinkers | Purdue University |
| Landon Bundy | Rose-Hulman Institute of Technology |
| Junhao Chen | Rose-Hulman Institute of Technology |
| Evan Cochrane | Rose-Hulman Institute of Technology |
| Jess Corso | Rose-Hulman Institute of Technology |
| Liam Enneking | Rose-Hulman Institute of Technology |
| Pierce Ferner | Rose-Hulman Institute of Technology |
| Luke Francisco | Purdue University |
| David Gaebler | Hillsdale College |
| Casey Garner | Rose-Hulman Institute of Technology |
| Kevin Gerstle | Hillsdale College |
| Jessica Gilbert | Purdue University |
| William Green | Rose-Hulman Institute of Technology |
| Torin Greenwood | Rose-Hulman Institute of Technology |
| Ralph Grimaldi | Rose-Hulman Institute of Technology |
| Croix Gyurek | IUPUI |
| Katy Gyurek | IUPUI |
| Andrea Hall | Aquinas College |
| Eric Hall | Aquinas College |
| Kelsey Hall | Aquinas College |
| Isaac Harper | Rose-Hulman Institute of Technology |
| Jordan-Taylor Harris | Purdue University |
| Menna Hassan | Purdue University |
| Megan Heyman | Rose-Hulman Institute of Technology |
| A J Hildebrand Hildebrand | University of Illinois |
| Thomas Hoevener | Rose-Hulman Institute of Technology |
| Joshua Holden | Rose-Hulman Institute of Technology |
| Allen Holder | Rose-Hulman Institute of Technology |
| Leanne Holder | Rose-Hulman Institute of Technology |
| Jiasheng Hu | Rose-Hulman Institute of Technology |
| Michael Ivanitskiy | University of Michigan |


| Alison Jeffries | Purdue University |
| :---: | :---: |
| Carlyn Johannigman | Rose-Hulman Institute of Technology |
| Hayley Jordan | Purdue University |
| Phillip Kerger | Fordham University |
| Jun Kim | Purdue University |
| Seonguk Kim | DePauw University |
| Ryan Kulwicki | Valparaiso University |
| Kali Lacy | Purdue University |
| Kevin LaMaster | Purdue University |
| Jayla Langford | Purdue University |
| Kevin Lewis | Rose-Hulman Institute of Technology |
| Charles Li | Mercy College |
| Xiaomin Li | University of Illinois |
| Zeyu Liao | Rose-Hulman Institute of Technology |
| Kendra Lin | Purdue University |
| Margaret Luffman | Rose-Hulman Institute of Technology |
| Bochuan Lyu | Rose-Hulman Institute of Technology |
| Namaluba Malawo | Purdue University |
| Michael McDaniel | Aquinas College |
| John McSweeney | Rose-Hulman Institute of Technology |
| Chandler Meyers | University of Michigan |
| Igor Minevich | Rose-Hulman Institute of Technology |
| Rachel Morris | University of Richmond |
| Garrett Mulcahy | Purdue University |
| Jonathan Myers | Rose-Hulman Institute of Technology |
| Tyler Netherly | Purdue University |
| Son Nguyen | DePauw University |
| Madelyn Nolting | IUPUI |
| Reed Phillips | Rose-Hulman Institute of Technology |
| Abigail Price | Hillsdale College |
| Dave Rader | Rose-Hulman Institute of Technology |
| Kelli Rice | DePauw University |
| John Rickert | Rose-Hulman Institute of Technology |
| Manda Riehl | Rose-Hulman Institute of Technology |
| Leonardo Rodriguez Gutierrez | University of Illinois |
| Patricia Salas | University of Central Oklahoma |
| Kelvin Saldana | Mercy College |
| Kathryn Schantz | University of Kentucky |
| Haydn Schroader | Purdue University |
| Rose Schweizer | Hillsdale College |
| I'Yanna Scott | Purdue University |
| Jason Selbo | Purdue University |
| Yosi Shibberu | Rose-Hulman Institute of Technology |


| Samantha Smock | Purdue University |
| :--- | :--- |
| Jose Smokowski | Rose-Hulman Institute of Technology |
| Sarthak Suri | Rose-Hulman Institute of Technology |
| Seiiji Takagi | Rose-Hulman Institute of Technology |
| Xin Tang | Rose-Hulman Institute of Technology |
| Wayne Tarrant | Rose-Hulman Institute of Technology |
| Brandon Tate | Rose-Hulman Institute of Technology |
| Lee Trent | Rose-Hulman Institute of Technology |
| Sang Truong | DePauw University |
| Dylan Verst | Rose-Hulman Institute of Technology |
| Angela Vichitbandha | University of Kentucky |
| Jiawen Wang | Rose-Hulman Institute of Technology |
| Mark Daniel Ward | Purdue University |
| Tracy Weyand | Rose-Hulman Institute of Technology |
| Allen Williams | University of Toledo |
| Yu Xin | Rose-Hulman Institute of Technology |
| Congwei Yang | University of Illinois |
| Wojciech Zacherek | Rose-Hulman Institute of Technology |
| Zheng Zhang | Rose-Hulman Institute of Technology |
| Muqing Zheng | Rose-Hulman Institute of Technology |

