Rose-Hulman Institute of Technology – 24'th Undergraduate Mathematics Conference
Student Speaker Abstracts; Listed according to Presentation Times

Friday Program * = *presenter*

Friday, 3:30 - 3:50 p.m., G-219 Crapo Hall
Normal Subgroups of a Wreath Product of a Finite p-Group and Corresponding Doubly-Invariant Vector Subspaces: Preliminary Report
Sarah R. Bockting* (University of Evansville), Arran C. Hamm (Wake Forest University), Sam Ruth (Northwestern University)

Abstract: This project was part of a REU program at the University of Akron. Before our work began with this project, our project advisor, Dr. Jeff Riedl, had previously established a correspondence between the normal subgroups of finite p-groups arising from wreath products and certain matrix subspaces that are invariant under two specific transformations. These transformations can be thought of as two partial derivatives on a function space from Z/pZ x Z/pZ to Z/pZ. When thought of as transformations on matrix spaces, one can algorithmically find all the doubly-invariant subspaces. We investigated the p=5 case in order to discover both the number and the form of these subspaces, which is equivalent to counting and describing the normal subgroups of these wreath products. The first step in tackling this problem was to split it into cases, based on the number and arrangement of standard basis matrices contained in each doubly-invariant subspace. This splits the general problem into 2p C p cases, which for p=5, is 252. Some of these cases are quite trivial to work with, while others are quite complicated. The presentation will illustrate the general method used for working through the different cases and discuss examples that display the range of this problem.

Friday, 3:30 - 3:50 p.m., G-221 Crapo Hall
The Count is Plus Eight. Should I Hit, Split, Double Down, or Stay??
Diem-Trang Le, Siena Heights University

Abstract: Blackjack is one of the most popular card games at any casino. Over time, the way Blackjack is played has changed to make the game more favorable to the casino. This project will take an in depth look at Blackjack and the various playing strategies to increase winnings. In particular, this study will explore the effects of card counting as a factor in increasing the probability of winning. Based on may card counting techniques, the study will be using the HI-LOW card counting technique to see if this will improve winnings.

Friday, 4:00 - 4:20 p.m., G-219 Crapo Hall
Galois Representations Associated to Modular Forms
Hari Ravindran, Rose-Hulman Institute of Technology

Abstract: Classically, a modular form is a meromorphic function on the upper complex half-plane which exhibits certain symmetry properties. As such, it has a Fourier expansion whose coefficients contain a wealth of arithmetic data. Specifically, a theorem of Shimura* (generalized by Deligne and Serre) associates an elliptic curve to each modular form with rational coefficients. Torsion subgroups of the associated elliptic curve then give rise to Galois representations, whose traces and determinants (usual notions) are related to the Fourier coefficients. In this talk we will explicitly give the Fourier expansion of a certain modular form, partially compute the associated representation, and verify that the theorem holds in this case. Familiarity with basic abstract algebra will be useful but not necessary to understand the talk.

Friday, 4:00 - 4:20 p.m., G-221 Crapo Hall
Determinants of Aggregate Saving: Analysis of Lagged Effects from U.S. Data, 1959-2006
Kathryn Peters and Miranda Voegel, University of Evansville

Abstract: According to the Federal Reserve Bank of St. Louis, Americans have saved a negative percentage of their incomes every month since April 2005, reflecting a recent trend in American habits to save an increasingly smaller proportion of after-tax income. This trend suggests a need to reexamine the relationship between saving and its theoretical determinants, which economic theory suggests include income and interest rates. We captured nearly 50 years of the savings trend in America using a combination of real disposable personal income, the federal funds rate, and the unemployment rate. Utilizing LIMDEP statistical software and a lesser-known distributed lag technique, our model also demonstrates evidence that suggests past data have a statistically significant impact on the current savings rate.

Friday, 4:40 - 5:00 p.m., G-219 Crapo Hall
A Mathematical Algorithm for the Thermal Imaging of Defects
Victor B Oyeyemi, Goshen College

Abstract: Monitoring objects for defect is an issue that is interesting to both Mathematicians and Engineers alike. In industrials settings, for example, the ability to effectively detect the presence and the nature of internal defects in machineries, vats, furnaces, and so on is often very important for reasons of safety, cost reduction, and quality assurance. This talk is about a mathematical algorithm capable of detecting the presence of spherical void(s) in a thermally conducting object. The algorithm is capable of recovering both the radii and locations of each void. Our method involves the application of a known steady-state heat flux to the object's boundary. Steady state temperature measurements from the boundary is then used as data for reconstructing the void(s) hidden within the object. Positive results were obtained from simulations designed to test the algorithm.

Friday, 4:40 - 5:00 p.m., G-221 Crapo Hall
On the Minimum Vector Rank of a Graph
Ian Rogers, Rose-Hulman Institute of Technology

Abstract: Given a graph or multigraph G on n vertices, we associate a set of nonzero complex vectors to the vertices of G in the following manner: If vertices i and j are not joined then the corresponding vectors are orthogonal, and if i and j are connected by a single edge, the associated vectors are not orthogonal. The rank of a vector representation is the maximum number of linearly independent vectors in the representation. The minimum vector rank of G, mvr(G), is the minimum rank among all vector representations of G.

We present methods for determining mvr(G) if G is among certain classes of graphs, including perfect graphs, complete graphs, and cycles. Further, we present upper and lower bounds on mvr(G) for all multigraphs that contain only multiedges, and provide two conjectures on the exact value of mvr(G) for a graph.

Saturday Program * = *presenter*

Saturday, 10:10 - 10:30 a.m., G-219 Crapo Hall
Computer Lease/Purchase Options for the Fortune 500
Bryan Beebe, Siena Heights University

Abstract: Computer technology changes rapidly. It seems like as soon as you get new technology it is already out of date. Keeping this in mind, we look at a fortune 500 company as they try to decide to lease or buy their computer equipment. A mathematical model will be developed to attempt to minimize cost, without negatively impacting productivity, using variables such as length of lease, depreciation on computer equipment, tax rebates on depreciation, and monthly payment cost vs. total cost of new computers.

Saturday, 10:10 - 10:30 a.m., G-221 Crapo Hall
Novel Statistical Method for Analyzing Spectroscopic Ligand Binding Data Using Matlab
Adam Gouge, Truman State University

Abstract: The estrogen receptor protein plays a key role in the development of breast cancer. In order to learn more about the disease and its treatment, the effect of ligands such as the commonly prescribed anti-cancer pharmaceutical, tamoxifen on the receptor protein must be understood. By measuring quenching of intrinsic tryptophan fluorescence due to fluorescence resonance energy transfer, the amount of ligand bound to the receptor can be determined. However, the concentration of unbound ligand cannot be measured directly; therefore a plot of bound versus unbound ligand concentration does not exhibit a true independent variable. Attempts at using least squares non-linear regression techniques for standard equations of ligand binding yielded fitting parameters that were highly dependent on the initial values chosen. Furthermore, this method frequently resulted in physically impossible fitting parameters. The standard equation for ligand binding was solved for total ligand concentration as a function of the bound concentration. Using the equation derived and a highly iterative least squares optimization method written in Matlab, unique and physically possible solutions for the Hill coefficient and K0.5 fitting parameters were obtained. Identical values for the fitting parameters were obtained regardless of the initial values used. Analysis using this method showed that the human estrogen receptor exhibited positively cooperative binding to tamoxifen. The Hill coefficient observed was 1.41 ± 0.07 and the K0.5 was 0.024 ± 0.006 μM, consistent with literature values. The method described can be used to analyze data from any receptor binding assay in which only the concentration of bound ligand can be measured directly.
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Saturday, 10:40 - 11:00 a.m., G-219 Crapo Hall
Affinely Self-Generating Sets and Morphisms
Adam Gouge, Truman State University
Abstract: Kimberling defined a self-generating set S of integers as follows. Assume 1 is a member of S, and if x is in S, then 2x and 4x-1 are in S. We study similar self-generating sets of integers whose generating functions come from a class of affine functions for which the coefficients of x are powers of a fixed base. We prove that for any positive integer m, the resulting sequence, reduced modulo m, is the image of an infinite word that is the fixed point of a morphism over a finite alphabet. We also prove that the resulting characteristic sequence of S is the image of the fixed point of a morphism of constant length, and is therefore automatic.

Saturday, 10:40 - 11:00 a.m., G-221 Crapo Hall
A Look at the Mathematics of Automated Model Checking
Brandon Borkholder, Rose-Hulman Institute of Technology
Abstract: Formal software verification is gaining acceptance in the industry, but there remains much to be done before automated model checking is efficient enough to be widely adopted. Model checking is used to verify software conformity to some specification (requirements). Over the past few decades there have been many methods used to implement automated model checking. These algorithms vary from computing the entire state space to solving a related satisfiability problem. We will discuss some of the most popular methods and the mathematics behind them.

Saturday, 11:20 - 11:40 a.m., G-219 Crapo Hall
Refinability of Rational Functions
Ely Spears, Rose-Hulman Institute of Technology

Abstract: A k-refinable function is a function f(x) that can be re-written in terms of the function f(kx). In recent decades, refinable functions have become increasingly popular due to their desirable properties in many applications, such as wavelet analysis. While the refinability properties of many popular classes of functions, such as compactly supported splines, have been known for a while, rational functions had seemed to escaped notice in terms of refinability. This talk is based on research investigating the refinability of rational functions that took place at Texas A&M University during the summer. Preliminary simplifications to the general problem are presented in a chronological collection of lemmas. A complete characterization of refinable rational functions follows with an interesting connection to an open problem in number theory.

Saturday, 11:20 - 11:40 a.m., G-221 Crapo Hall
The Winning Hand
Brad Hubartt, University of Evansville

Abstract: Poker, along with any other gambling game, is governed by specific probabilities. Since money is often at stake, knowing the outcome before anyone else is a huge advantage. Using conditional probabilities we wrote a computer program that returns the odds of any hand winning winning when we are competing with just one other person. This computer program applies solely to Texas Hold'em Poker. Very few modest assumptions were made by us when writing the program, which only have a slight affect on the results.

Saturday, 11:50 - 12:10 a.m., G-219 Crapo Hall
Characterizing Holes in Wires and Plates: Inverting the Heat Equation
Thomas Werne, Rose-Hulman Institute of Technology

Abstract: The heat equation is a classical partial differential equation that predicts the temperature distribution on a region with given boundary conditions. It is interesting to examine the "inverse problem," namely, by looking only at the boundary data (i.e. the temperature on the outside of a plate and heat flow into it), can we determine what the region looks like? Specifically, can we find holes in the region? This talk will define the inverse problem, discuss techniques used to solve it, and present results based on work done during the summer of 2006 by the Inverse Problems REU group of Thomas Werne (Rose-Hulman) and Jay Preciado (The College of New Jersey) at Rose-Hulman under Dr. Bryan (Mathematics Dept. Rose-Hulman).

Saturday, 11:50 - 12:10 a.m., G-221 Crapo Hall
Optimizing 4th-Order and 5th-Order Explicit Runge-Kutta Formulas
Stephen Dupal, Rose-Hulman Institute of Technology

Abstract: Differential equations have been solved numerically with explicit Runge-Kutta methods for over a century. Runge-Kutta methods are used in the sciences as well as mathematical software such as Matlab's ode45 solver. Utilizing techniques in polynomial theory based on Gröbner bases, it becomes more manageable to find Runge-Kutta formulas that minimize higher-order truncation error. In this talk, we will discuss the connection between the Runge-Kutta method and Gröbner bases, and we will present some of the results of exploring the optimization of fourth- and fifth-order Runge-Kutta formulas. This presentation is based on work done by Iowa State University's summer 2006 Numerical Analysis REU group consisting of Stephen Dupal (Rose-Hulman) and Michael Yoshizawa (Pomona College).

***We gratefully acknowledge support for this year's conference from funding through the MAA and the NSF grant* DMS-0536991.**