

Normal Calculations

Since the normal density cannot be integrated in closed form we must compute normal probabilities numerically (using software or a calculator) or using a standard normal table.

To compute normal probabilities involving a normal random variable W with mean μ_w and standard deviation σ_W using a standard normal table we follow a two-step procedure:

- i. We convert the problem to one involving a standard normal RV by standardizing:
 1. We subtract the mean μ_W from appropriate quantities.
 2. We divide appropriate quantities by the standard deviation σ_W .
 3. We use the fact that $(W - \mu_W)/\sigma_W = Z$ where Z is standard normal.
- ii. We compute standard normal probabilities using a normal table (e.g., Table III) which provides values of the standard normal CDF $F_Z(z)$ (denoted by $\Phi(z)$) using the fact that

$$\begin{aligned}P(a < Z < b) &= F_Z(b) - F_Z(a) \\ &= \Phi(b) - \Phi(a)\end{aligned}$$

Example 1: A popular IQ measurement is normally distributed with a mean of 100 and a standard deviation of 15. Do the following:

- i. Determine the fraction of people with an IQ between 85 and 115.
- ii. A person with an IQ of 160 or more is considered a genius. Determine the fraction of people who are geniuses according to this criterion.
- iii. *Mensa* is a high IQ social club. To be in *Mensa* your IQ (as measured by some standardized test) must be in the top 2%. Using this IQ measurement, what's the smallest a person can score and be eligible for *Mensa*?

Example 2 (223 Exam Question): Let F be the volume of cola put into 20 oz bottles by a filling machine. Suppose F is normal with standard deviation $\sigma_F = 0.2$ oz and mean $\mu_F = 20.5$ oz.

- i. A bottle is underfilled if the fluid content is less than 20 oz. For the current setting of the machine, what fraction of the bottles are underfilled?
- ii. In order to reduce the fraction of underfilled bottles to 0.2%, σ_F must be reduced to a new value, σ'_F . Determine the value of σ'_F which yields 0.2% underfilled.