

Regression Random Error Variance and Regression Coefficient Standard Errors

1: For the bivariate data $\{ (-2, -1), (0,3), (2,1) \}$ do the following:

- i.** Manually compute the least squares estimates $\hat{\beta}_0$ and $\hat{\beta}_1$ using the formula $\hat{\beta} = (X'X)^{-1}X'\vec{Y}$.
- ii.** Check you results in part i using Minitab's regression routine

Stat -> Regression -> Regression -> Fit Regression Model

iii. Manually compute the residuals

$$\begin{aligned}e_i &= y_i - \hat{y}_i \\ &= y_i - (\hat{\beta}_0 + \hat{\beta}_1 x_i)\end{aligned}$$

and then compute the least squares estimate of the random error variance σ^2 :

$$\hat{\sigma}^2 = \frac{1}{n-2} \sum_{i=1}^n e_i^2$$

Compare the square root of this, $\hat{\sigma}$, with the value labeled "S" in your Minitab regression output. They should be the same.

- iv.** Compute the matrix $\hat{\sigma}^2(X'X)^{-1}$ then calculate the square roots of the diagonals to get the standard deviations (standard errors) of $\hat{\beta}_0$ and $\hat{\beta}_1$. Compare your results with the values in the SE Coef column in the Minitab regression coefficient table.