MA 381, Quiz 1, Solutions for Problems 2, 3, and 4

2: Let A, B be two events in the same sample space such that $P(A) = 0.8$ and $P(AB) = 0.6$.

i. (1 pt.) A and B are mutually exclusive. Circle one: **True** or **False**.

Since $P(AB) = 0.6 > 0$, both events can occur in one execution of the experiment so they are not mutually exclusive.

ii. (3 pts.) Determine the maximum possible value for $P(B)$.

$$\begin{aligned} 1 &\geq P(A \cup B) \\ &= P(A) + P(B) - P(AB) \\ &= 0.8 + P(B) - 0.6 \\ &= 0.2 + P(B). \end{aligned}$$

Since $1 \geq 0.2 + P(B)$, $0.8 \geq P(B)$.

3: (4 pts.) Determine the number of permutations of *BESERKELEY*.

Since the 4 E's are not distinguishable, 10! overcounts the number of permutations by 4!. Thus the number of permutations is 10!/4!. Recall the discussion of the number of permutations of *BERKELEY* in class and on page 49 in our book.

4: (4 pts.) Determine the probability of getting at least one king in a 5-card hand drawn from a normal deck of 52 cards. Assume the deck is well-shuffled and the cards are drawn, as usual, without replacement.

By the complement rule, P("at least one king") = 1 - P("no kings"). Since there are 48 cards in the deck which are not kings, there are ${}_{48}C_5$ 5-card hands with no kings. Since there are ${}_{52}C_5$ equally-likely 5-card hands, we see that

$$P(\text{“at least one king”}) = 1 - \frac{\binom{48}{5}}{\binom{52}{5}}$$