#4 (#1, pg. 433)

$$Cov(X,Y) = \rho(X,Y)\,\sigma_X\,\sigma_Y = \tfrac{1}{2}(2)(3) = 3$$

$$\therefore\ Var(2X-4Y+3) = Var(2X-4Y)$$

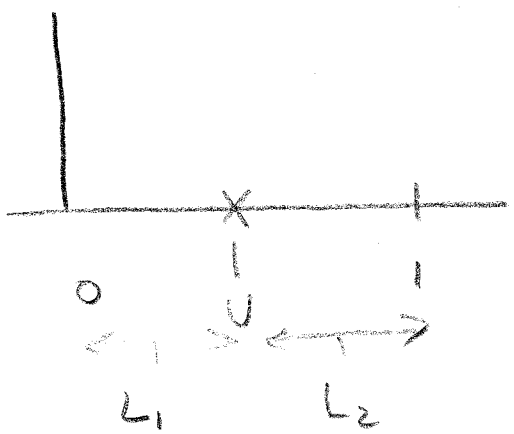
$$= 2^2\,Var(X) + (-4)^2\,Var(Y) + 2(2)(-4)\,Cov(X,Y)$$

$$= 4(2^2) + 16(3^2) - 16(3) = \underline{112}$$

#5 (#3, pg. 433)

L_1 = length 1st piece
$= U$

L_2 = length 2nd piece
$= 1-U$



where U uniform $(0,1)$

$$E[L_1] = E[U] = \tfrac{1}{2}$$

$$E[L_2] = E[1-U] = 1 - E[U] = \tfrac{1}{2}$$

$$E[L_1 L_2] = \int_0^1 u(1-u)\,du = \left(\frac{u^2}{2} - \frac{u^3}{3}\right)\Big|_0^1 = \frac{1}{6}$$

$$\therefore\ \boxed{Cov(L_1,L_2) = \frac{1}{6} - \frac{1}{2}\left(\frac{1}{2}\right) = \frac{4}{24} - \frac{6}{24} = -\frac{1}{12}}$$