#3 (#15, pg. 426) Cont.:

$$V_1 = \frac{1}{2}(V+W) = \frac{1}{2}V + \frac{1}{2}W$$

$$\therefore Var(V_1) = \left(\frac{1}{2}\right)^2 Var(V) + \left(\frac{1}{2}\right)^2 Var(W) + 2\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) Cov(V,W)$$

$$= \frac{1}{4} Var(V) + \frac{1}{4} Var(W) + 0$$

since V, W independent measurements

$$= \frac{1}{4}\sigma^2 + \frac{1}{4}\sigma^2 = \frac{\sigma^2}{2}$$

Similarly

$$V_2 = \frac{1}{2}(V-W) = \frac{1}{2}V - \frac{1}{2}W$$

$$\therefore Var(V_2) = \left(\frac{1}{2}\right)^2 Var(V) + \left(\frac{-1}{2}\right)^2 Var(W) + 2\left(\frac{1}{2}\right)\left(\frac{-1}{2}\right) Cov(V,W)$$

$$= \frac{1}{4} Var(V) + \frac{1}{4} Var(W) + 0$$

$$= \frac{\sigma^2}{2}$$

Therefore, although we still only used two measurements, we halved the variance of our estimates.