MA 381 HW 16

#2 (#18, pg. 426) [6 pts.]

a) $Var(X+Y) = Var(X) + Var(Y) + 2\,Cov(X,Y)$

$$E[X] = \int_0^1 \int_x^1 x \cdot 8xy\,dy\,dx = \frac{8}{15} \quad \} 1$$

$$E[Y] = \int_0^1 \int_x^1 y \cdot 8xy\,dy\,dx = \frac{4}{5} \quad \} 1$$

$$E[XY] = \int_0^1 \int_x^1 xy \cdot 8xy\,dy\,dx = 4/9 \quad \} 1$$

$$Cov(X,Y) = E[XY] - E[X]E[Y] = \frac{4}{9} - \frac{8}{15}\left(\frac{4}{5}\right) = \frac{4}{225}$$

1

$$Var(X) = E[X^2] - [E(X)]^2 = 1/3 - \left(\frac{8}{15}\right)^2 = \frac{11}{225}$$

$$Var(Y) = E[Y^2] - [E(Y)]^2 = 2/3 - \left(4/5\right)^2 = \frac{2}{75}$$

$$\therefore Var(X+Y) = \frac{11}{225} + \frac{2}{75} + 2\left(\frac{4}{225}\right) = \underline{\frac{1}{9} \doteq .111} \quad \} 1$$