MA 381 ITW 13 Solutions

#7 (#5, pg. 474)

$Y = X_1 + X_2 + \cdots + X_n$ where the X_i are independent gamma with parameters r_i and λ.

$$M_{X_i}(t) = \left(\frac{\lambda}{\lambda - t}\right)^{r_i}$$

$$M_Y(t) = M_{X_1}(t) \cdots M_{X_n}(t)$$

$$= \left(\frac{\lambda}{\lambda - t}\right)^{r_1} \cdots \left(\frac{\lambda}{\lambda - t}\right)^{r_n}$$

$$= \left(\frac{\lambda}{\lambda - t}\right)^{r_1 + r_2 + \cdots + r_n}$$

$\therefore$ Y is gamma with parameter λ and $r = r_1 + r_2 + \cdots + r_n$.