MA 381 HW 13 Solutions

#4 (#12, pg. 466)

$$M_X(t) = \frac{1}{1-t}, \quad t < 1$$

$$Y = 2X + 1$$

$$M_Y(t) = E[e^{tY}] = E[e^{t(2X+1)}] = e^t E[e^{t^*X}]$$

where $t^* = 2t$

$$= e^t M_X(t^*)$$

$$= e^t \frac{1}{1-2t} = \underline{\frac{e^t}{1-2t}, \quad t < \tfrac{1}{2}}$$

(7 pts.)

#5 i. $M_X(t) = \frac{1}{2}e^{-t} + \frac{1}{2}e^{t}$ } 1

ii. $M_Y(t) = M_X(t) \cdot M_X(t)$

$$= \left(\tfrac{1}{2}e^{-t} + \tfrac{1}{2}e^{t}\right)^2 = \underbrace{\tfrac{1}{4}e^{-2t} + \tfrac{1}{2} + \tfrac{1}{4}e^{2t}}_{\text{2}}$$

$$\therefore \quad P_Y(y) = \begin{cases} 1/4, & x = -2 \\ 1/2, & x = 0 \\ 1/4, & x = 2 \\ 0, & \text{otherwise} \end{cases} \quad \text{1}$$