MA 381 HW 13 Solutions

$$\frac{12(1-e^t)}{t^3} = 12\left[\frac{1}{t^3}\left(-t - \frac{t^2}{2!} - \frac{t^3}{3!} - \frac{t^4}{4!} - \cdots\right)\right]$$

$$= -12t^{-2} - 6t^{-1} - \frac{12}{3!} - \frac{12}{4!}t^1 - \frac{12}{5!}t^2 - \cdots$$

$$\frac{6(1+e^t)}{t^2} = \frac{6}{t^2}\left(2 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} + \cdots\right)$$

$$= 12t^{-2} + 6t^{-1} + \frac{6}{2!} + \frac{6}{3!}t + \frac{6}{4!}t^2 + \cdots$$

$$M_X(t) = \frac{6}{2!} - \frac{12}{3!} + \left(\frac{6}{3!} - \frac{12}{4!}\right)t + \left(\frac{6}{4!} - \frac{12}{5!}\right)t^2 + \cdots$$

$$= 1 + \frac{E[X]}{1!}t + \frac{E[X^2]}{2!}t^2 + \cdots$$

$$\therefore\ E[X] = \frac{6}{3!} - \frac{12}{4!} = \frac{6}{6} - \frac{12}{24} = \underline{\underline{1/2}}$$

Alternatively, they could differentiate $M_X(t)$ to get $M_X^{(1)}(t)$ and then use

$E[X] = M_X^{(1)}(0) = 1/2$ by L'Hospital's Rule.