MA 381 HW 13 Solutions

#3 (#5, pg. 465)

$$M_X(t) = E[e^{tX}] = \int_0^1 e^{tx} \cdot 6x(1-x)\,dx$$

$$= 6\int_0^1 xe^{tx}dx - 6\int_0^1 x^2e^{tx}dx$$

$$\int_0^1 x^2e^{tx}dx \qquad du = e^{tx}dx \quad U = \frac{1}{t}e^{tx}$$
$$V = x^2 \qquad dv = 2x\,dx$$

$$= \frac{x^2}{t}e^{tx}\Big|_0^1 - \frac{2}{t}\int_0^1 xe^{tx}dx$$

$$= \frac{1}{t}e^t - \frac{1}{t}\int_0^1 xe^{tx}dx$$

$$\therefore M_X(t) = \left(6 + \frac{12}{t}\right)\int_0^1 xe^{tx}dx - \frac{12}{t}e^t$$

$$= 6\left(1+\frac{2}{t}\right)\left[\frac{e^t}{t} - \frac{e^t}{t^2} + \frac{1}{t^2}\right] - \frac{12}{t}e^t$$

$$= \begin{cases} \dfrac{12(1-e^t)}{t^3} + \dfrac{6(1+e^t)}{t^2}, & t \neq 0 \\ 1, & t = 0 \end{cases}$$