MA 381 HW 13 Solutions

\# 1. (#3, pg. 465)

$$E[e^{tX}] = \sum_{j=1}^{\infty} e^{tj}\, 2\left(\tfrac{1}{3}\right)^j = \sum_{j=1}^{\infty} 2\left(\tfrac{e^t}{3}\right)^j$$

$$= \frac{2e^t}{3}\left[1 + \left(\frac{e^t}{3}\right)^1 + \left(\frac{e^t}{3}\right)^2 \cdots\right]$$

$$= \frac{2e^t}{3} \cdot \frac{1}{1 - e^t/3} = \frac{2e^t}{3 - e^t}, \quad \frac{e^t}{3} < 1$$

or $t < \ln 3$

$$E[X] = M_X^{(1)}(0)$$

$$M_X^{(1)}(t) = \frac{d}{dt}\left\{ 2e^t (3-e^t)^{-1} \right\}$$

$$= -2e^t(3-e^t)^{-2}(-e^t)$$

$$+ 2e^t(3-e^t)^{-1}$$

$$= \frac{2e^{2t}}{(3-e^t)^2} + \frac{2e^t}{3-e^t}$$

$$\therefore E[X] = \frac{2}{(3-1)^2} + \frac{2}{3-1} = \frac{1}{2} + 1 = \underline{\underline{3/2}}$$