5: A random (IID) sample of $n = 25$ observations of a non-normal RV X is to be acquired. Let $\bar{X}$ represent the average (sample mean) of these measurements once they are acquired. Suppose the mean of X is 100 and the standard deviation is 25. Do the following:

i. (2 pts.) What is the mean of $\bar{X}$, $\mu_{\bar{X}}$? $\mu_{\bar{X}} = \mu_X = 100$

ii. (2 pts.) What is the standard deviation of $\bar{X}$, $\sigma_{\bar{X}}$? $\sigma_{\bar{X}} = \frac{\sigma_X}{\sqrt{n}} = \frac{25}{\sqrt{25}} = 5$

iii. (4 pts.) What is the approximate probability that $\bar{X}$ will be between 98 and 102?

$$P(98 \le \bar{X} \le 102) = P\left(\frac{98-100}{5} \le Z_1 \le \frac{102-100}{5}\right)$$

$$= P(-.4 \le Z \le .4)$$

$$\begin{array}{r} .6554 \\ -\ .3446 \\ \hline .3108 \end{array}$$

$$= \Phi(.4) - \Phi(-.4) = .6554 - .3446 = \underline{.3108}$$

6: (5 pts.) RV W has moment-generating function $M_W(t) = (1-2t)^{-1/2}$, $t < 1/2$. Compute E[W].

$$M_W'(t) = -\tfrac{1}{2}(1-2t)^{-3/2} \cdot (-2)$$

$$= (1-2t)^{-3/2}$$

$$E[W] = M_W'(t)\Big|_{t=0} = (1-2(0))^{-3/2} = \underline{1}$$