

4: Let  $X$  and  $Y$  have the joint density function

$$f_{X,Y}(x,y) = \begin{cases} 2, & 0 \leq y \leq x \leq 1 \\ 0, & \text{elsewhere} \end{cases}$$

i. (5 pts.) Compute the marginal density of  $X$ ,  $f_X$ .

$$f_X(x) = \int_0^x 2 dy, \quad 0 < x \leq 1$$

$$= \begin{cases} 2x, & 0 < x \leq 1 \\ 0, & \text{elsewhere} \end{cases}$$

$$f_Y(y) = \int_y^1 2 dx$$

$$= 2x \Big|_y^1$$

$$= 2 - 2y$$

ii. (5 pts.) Compute the conditional probability density function of  $Y$  given  $X = x$ ,  $f_{Y|X}(y|x)$ .

$$f_{Y|X}(y|x) = \frac{f_{X,Y}(x,y)}{f_X(x)}, \quad f_X > 0$$

$$= \frac{2}{2x} = \begin{cases} \frac{1}{x}, & 0 < y \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

iii. (2 pts.) Are  $X$  and  $Y$  independent? Justify your answer for full credit.

NO.  $f_{X,Y}$  is non-zero on non-rectangular range.