2: (5 pts.) The RV X has density function

$$f_X(x) = \begin{cases} 2x, & 0 \le x \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

Compute the density function f_Y of the RV $Y = X^2$.

$$F_Y(t) = P(Y \le t) = P(X^2 \le t)$$
$$= P(X \le \sqrt{t})$$
$$= F_X(\sqrt{t})$$

$$\therefore f_Y(t) = f_X(\sqrt{t}) \cdot \tfrac{1}{2} t^{-1/2} = 2(\sqrt{t}) \cdot \tfrac{1}{2} \tfrac{1}{\sqrt{t}} = 1$$

$$= \begin{cases} 1, & 0 \le t \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

3: (5 pts.) The lifetimes of a certain type of bulb are exponential with a mean lifetime of 2 years. Give an expression (integral, sum, whatever) - **but do not evaluate/simplify it** - for the probability that a bulb of this type lasts more than 1 year.

$$\mu = \frac{1}{\lambda} = 2 \quad \therefore \lambda = 1/2$$

$$P(\text{Lifetime} \ge 1) = \int_1^\infty \tfrac{1}{2} e^{-\frac{1}{2}x} \, dx = e^{-1/2}$$