2: Consider the RV Y with the following density function:

$$f_Y(y) = \begin{cases} \frac{3}{2}y^2, & -1 \le y \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

Do the following. If you need more room use the back of this page.

i. (4 pts.) Determine the CDF of Y, F_Y, for the entire real line.

$$F_Y(t) = \begin{cases} 0, & t < -1 \\ \frac{1}{2}t^3 + \frac{1}{2}, & -1 \le t \le 1 \\ 1, & t > 1 \end{cases}$$

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ii. (2 pts.) Compute $P(0 < Y < 1)$.

$$P(0 < Y < 1) = F_Y(1) - F_Y(0)$$

$$= \frac{1}{2} \cdot 1 + \frac{1}{2} - \frac{1}{2} = 1/2$$

iii. (6 pts.) Compute μ_Y and σ_Y^2.

$$\mu_Y = \int_{-1}^{1} y \cdot \frac{3}{2} y^2 \, dy = \int_{-1}^{1} \frac{3}{2} y^3 \, dy = \frac{3}{8} y^4 \Big|_{-1}^{1} = 0 \Big\} \; 2$$

$$\sigma_Y^2 = E[Y^2] - \mu_Y^2$$

$$\underbrace{= E[Y^2]}_{2}$$

$$E[Y^2] = \int_{-1}^{1} y^2 \left(\frac{3}{2}\right) y^2 \, dy$$

$$= \frac{3}{10} y^5 \Big|_{-1}^{1} = \frac{3}{10}[1 - (-1)]$$

$$= \frac{3}{5} \Big\} \; 2$$

$$\therefore \sigma_Y^2 = 3/5 = 0.6$$