Disco II - Test #3

Professor Broughton Jan 21, 1999

Name:_____ Box #_____

Instructions

- Answer all the questions directly on the test.
- Calculator allowed
- Show all the necessary work and write your answers out neatly in English sentences.

Question	Possible Points	Points Obtained
1	10	
2	20	
3	20	
4	10	
5	20	
6	20	
Total	100	

1. Rook polynomials

Three tasks are to be assigned to 5 employees, with no employee getting more than one task. Task T_3 must not be given to employees E_1 or E_2, nor should T_1 be given to employee E_4.

1.a Put an X in the forbidden positions in the chart and calculate the rook polynomial of the forbidden positions.

	E_1	E_2	E_3	E_4	E_5
T_1					
T_2					
T_3					

1.b Find if the assignment can be made by calculating the number of ways the task assignment can be made, satisfying all the restrictions.

2. Partition Functions

Suppose we are selling gadgets singly, bundled into a pack 5 or a pack of 12. Find the generating function that describes the number of ways we can distribute n gadgets into singles, 5-packs and 12-packs. Write your answer as a rational function.

3. Generating Functions and Partial Fractions

Suppose we want to find the coefficient of x^{10} in $\frac{2x+x^2}{(1-x)^2(1+x)^2}$.

3.a Write out in series form:

$$\frac{1}{1-x} = \qquad , \frac{1}{(1-x)^2} =$$

$$\frac{1}{1+x} = \qquad , \frac{1}{(1+x)^2} =$$

3.b Write out $\frac{2x+x^2}{(1-x)^2(1+x)^2}$ in partial fraction form. Do not solve for the coefficients.

$$\frac{2x+x^2}{(1-x)^2(1+x)^2} =$$

3.c What system of equations must be solved to find the coefficients. Again, do not solve for the unknowns.

3.d Find the coefficient of x^{10} in $\frac{2x+x^2}{(1-x)^2(1+x)^2}$ in terms of the unknown coefficients, discussed in 3.*c*.

3.e By rewriting the generating function is there a simpler way to calculate the coefficient of x^{10}? What is the rewritten form?

4. Summatory convolution

Let $f(x)$ be the generating function for an unknown sequence $a_0, a_1, a_2, a_3, \ldots$. Find the generating function for the following sequences in terms of $f(x)$.

1. 4.a. $a_0, a_0 + a_1, a_0 + a_1 + a_2, a_0 + a_1 + a_2 + a_3 \ldots$

4.b. $a_0, a_1 - a_0, a_2 - a_1, a_3 - a_2 \ldots$

4.c. $\frac{a_0}{4}, \frac{a_0}{2} + \frac{a_1}{4}, \frac{a_0}{4} + \frac{a_1}{2} + \frac{a_2}{4}, \frac{a_1}{4} + \frac{a_2}{2} + \frac{a_3}{4}, \ldots$

5. Parity Question

5.a How many ternary (0,1,2)-sequences of ten symbols have an odd number of zeros.

5.b How many have an even number of zeros.

5.c True of false: The probability of an odd number of zeros is just about the same as the probability of an even number of zeros. Justify your answer.

6. Recurrence Equation

As in the example done in class, suppose we are trying to tile a $2 \times n$ chessboard with 2×1 dominos or a 2×2 squares. Suppose that the dominos are green and the squares are red or blue. Let a_n be the number of ways of tiling the chessboard, taking colour into account.

6.a Find

$$\begin{aligned} a_1 &= \\ a_2 &= \\ a_3 &= \end{aligned}$$

6.b Find a recurrence equation satisfied by the a_n, stating the initial conditions.