Disco II - Test #2

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Instructions

- Answer all the questions directly on the test.
- Calculator allowed
- Show all the necessary work and write your answers out neatly in English sentences.

Question	Possible Points	Points Obtained
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1. Topological sorting

Let S_1 be the divisors of 12 and S_2 be the divisors of 45.

1.a Draw the three Hasse diagrams of S_1, S_2 and $S_1 \cup S_2$ with the divisibility relation $x|y$. Identify the maximal, minimal, greatest and least elements in each diagram.

1.b Perform the topological sorting algorithm on $S_1 \cup S_2$ produce a total order. You need not draw the intermediate diagrams.

1.c Is the following true for all partial orderings, if not provide a counterexample: If $S_1, S_2 \subseteq S$ are such that both S_1 and S_2 have greatest elements then $S_1 \cup S_2$ has a greatest element.

2. Finite State Machines

Here is a next state and output table for a finite state machine on the binary alphabet $\mathcal{I} = \mathcal{O} = \{0, 1\}$.

ν	0	1
s_0	s_1	s_3
s_1	s_2	s_4
s_2	s_0	s_5
s_3	s_1	s_3
s_4	s_2	s_4
s_5	s_5	s_5

ω	0	1
s_0	0	0
s_1	0	0
s_2	1	0
s_3	0	0
s_4	0	0
s_5	0	0

2.a Minimize the machine by finding the successive partitions induced by 1-equivalence 2-equivalence and so on.

2.b Find a pair of inequivalent states whose minimizing string is a long as possible.

3. Dice and relations

Suppose we have six dice all of a different colour.

3.a What is the number the number of possible rolls in which all six different numbers appear?

3.b What is the number of tosses for which there are exactly three different numbers showing.

3.c Toss the dice. Declare that two colours are equivalent if the numbers on the dice are the same. Using the answer in 3.b, determine the of equivalence relations on six objects with at exactly three equivalence classes.

4. Inclusion exclusion

Consider the set of graphs on five vertices labelled v_1, v_2, v_3, v_4, v_5.

4.a Fill in the following table:

isolated vertices	max # edges	# graphs with given vertices isolated
none		1024
v_1		
v_1, v_2	3	
v_1, v_2, v_3		
v_1, v_2, v_3, v_4		
v_1, v_2, v_3, v_4, v_5		

4.b What proportion of graphs have no isolated vertices.?

5. Inclusion-Exclusion formulas

Recall the definitions:

$$\begin{aligned} S_0 &= N \\ S_1 &= \sum_i N(c_i) \\ S_2 &= \sum_{i<j} N(c_i c_j) \\ &\cdots \end{aligned}$$

$$\begin{aligned} E_m &= \#\{x : \text{exactly } m \text{ conditions hold}\} \\ L_m &= \#\{x : \text{at least } m \text{ conditions hold}\} \end{aligned}$$

Now let $n = 3$, the inclusion-exclusion formulas are:

$$\begin{aligned} E_0 &= S_0 - S_1 + S_2 - S_3 \\ E_1 &= S_1 - 2S_2 + 3S_3 \\ E_2 &= S_2 - 3S_3 \\ E_3 &= S_3 \end{aligned}$$

From these formulas find the formulas for the L_i terms of the S_i. Suggestion: first write out the L_i in terms of the E_i.