Disco II - Test #1

Professor Broughton Dec 16, 1999

Name:_____ Box #_____

Instructions

- Answer all the questions directly on the test.
- Show all the necessary work and write your answers out neatly in English sentences.

Question	Possible Points	Points Obtained
1	25	
2	25	
3	25	
4	25	
Total	100	

1. Languages

1.a Below there are two recursively defined languages and two languages defined by rules. In each of the languages find all words of length up to 4. All languages are subsets of $\{0,1\}^*$

A :

$$\begin{aligned} \lambda &\in A \\ \text{if } x &\in A \text{ then } 1x1, 0x0 \in A \end{aligned}$$

B :

$$\begin{aligned} &\lambda \in A \\ &\text{if } x \in A \text{ then } 1x1, 0x0, 0x1 \in A \end{aligned}$$

C :

$$\{x \in \{01\}^* : x^R = x, \|x\| \text{ is even}\}$$

D :

$$\{x \in \{01\}^* : \|x\| \text{ is even, } x \text{ has no more zeros than ones}\}$$

1.b Which languages are the same?

2. Finite State Machines

Here is a next state and output table for a finite state machine on the binary alphabet $\mathcal{I} = \mathcal{O} = \{0, 1\}$. It is a slight modification of the one on quiz.

ν	0	1
s_0	s_1	s_3
s_1	s_2	s_4
s_2	s_0	s_5
s_3	s_1	s_3
s_4	s_2	s_4
s_5	s_5	s_5

ω	0	1
s_0	0	0
s_1	0	0
s_2	1	0
s_3	0	0
s_4	0	0
s_5	0	0

2.a Draw the state diagram for this finite state machine.

s_0 s_1 s_2

s_3 s_4 s_5

2.b Are there any transient states, sinks or strongly connected proper submachines (with the same input alphabet)?

2.c Find all states reachable from s_1 in two steps. I.e., find all $s = \nu(s_1, x)$ where x has length 2.

3. Relations

Let $S = \{1, 2, 3, 4, 5, 6\}$, Let $\mathcal{R}_1$-denote the divisibility relation $x|y$ i.e., x exactly divides y. Let $\mathcal{R}_2$ denote the mod 3 congruence relation on $S, x \equiv y \bmod 3$

3.a Fill in the table with yes or no.

	$\mathcal{R}_1$	$\mathcal{R}_2$
reflexive		
symmetric		
anti symmetric		
transitive		
partial order		
equivalence		

3.b Pick one of the relations and fill in the relation matrix M below. (M is the zero-one submatrix that you fill in).

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

3.c Explain one of the properties of the relations in 3.a in terms of properties of the matrix M.

3.d Interpret the row sums of your matrix.

4. Numbers of relations

The set S is the same as in Question 3.

4.a How many reflexive relations are there?

4.b How many symmetric, reflexive relations are there?

4.c How many anti-symmetric reflexive relations are there?

4.d How many equivalence relations are partial orders?