Disco II - Quiz 6

Name:_____ Box #_____

In this quiz we will use the inclusion-exclusion principle to compute the Euler ϕ function of $120 = 2^3 \cdot 3 \cdot 5$. Let $S = \{1, 2, 3, \ldots, 120\}$. Define the following conditions:

c_1: the number $x \in S$ is divisible by 2
c_2: the number $x \in S$ is divisible by 3
c_3: the number $x \in S$ is divisible by 5

1. Compute these numbers:

$$\begin{aligned} N &= |S| = \\ N(c_1) &= \\ N(c_2) &= \\ N(c_3) &= \\ N(c_1c_2) &= \\ N(c_1c_3) &= \\ N(c_2c_3) &= \\ N(c_1c_2c_3) &= \end{aligned}$$

2. Using the information above compute:

$$N(\overline{c_1}\,\overline{c_2}\,\overline{c_3}) =$$

3. The Euler phi function of n is defined by

$$\phi(n) = |\{x : 1 \leq x \leq n, \gcd(x, n) = 1\}|.$$

Use the numbers computed in 2 and 3, in any way, to determine the numerical value of $\phi(120)$. Suggestion $\gcd(x, 120) = 1$ if and only if x and 120 have no common prime factors.

$$\phi(120) =$$