Disco II - Quiz 12

Name:_____ Box #_____

1. Recursion

Your borrow $\$10,000$ at an annual interest rate of 7% compounded monthly. The corresponding monthly compounding factor is given by

$$1 + \frac{0.07}{12} = 1.0058 = \alpha.$$

(FYI, the annualized multiplier is $\left(1 + \frac{0.07}{12}\right)^{12} = 1.0723$.) You propose to pay back β per month starting at the end of the first month.

1. Let P_n be the amount you owe at the beginning of the $n'th$ month (taking into account the previous months interest charge and the payment β has been made. Write out the recursion equation expressing P_{n+1} in terms of P_n, α and β. (no number just the symbols).

$$P_{n+1} = \alpha P_n - \beta.$$

2. Write out the first few terms P_1, P_2, P_3 expressed in terms of P_0, α and β.

$$\begin{aligned} P_1 &= \alpha P_0 - \beta, \\ P_2 &= \alpha^2 P_0 - (1+\alpha)\beta, \\ P_3 &= \alpha^3 P_0 - (1+\alpha+\alpha^2)\beta, \end{aligned}$$

3. Find a closed form for P_n.

$$\begin{aligned} P_3 &= \alpha^n P_0 - (1+\alpha+\alpha^2+\cdots+\alpha^{n-1})\beta, \\ &= \alpha^n P_0 - \frac{1-\alpha^n}{1-\alpha}\beta \end{aligned}$$

4. If you are to pay the loan off in 5 years, what should the payment β equal.

   We solve

$$\alpha^n P_0 - \frac{1-\alpha^n}{1-\alpha}\beta$$

for β :

$$\begin{aligned}\beta &= \alpha^n \frac{1-\alpha}{1-\alpha^n} P_0 \\ &= 10000 \left(1+\frac{0.07}{12}\right)^{60} \frac{\left(1+\frac{0.07}{12}\right)-1}{\left(1+\frac{0.07}{12}\right)^{60}-1} \\ &= 198.01\end{aligned}$$

Without interest the payment is

$$\frac{10000}{60} = 166.67$$

5. Suppose you are trying to tile a $3 \times n$ checkerboard with 1×2 dominos and 2×2. Let a_n be the number of distinct tilings. Find a_1, a_2, and a_3. Set up but do not solve the recurrence equation for a_n.

$$\begin{aligned}a_1 &= 0 \\ a_2 &= 6 \\ a_3 &= 0\end{aligned}$$

$$a_{n+2} = 5a_n$$

Cut a 3×2 piece of the end and figure out how many ways to tile this piece.