Disco II - Quiz 11

Name:_____ Box #_____

1. Rook Polynomials and forbidden positions

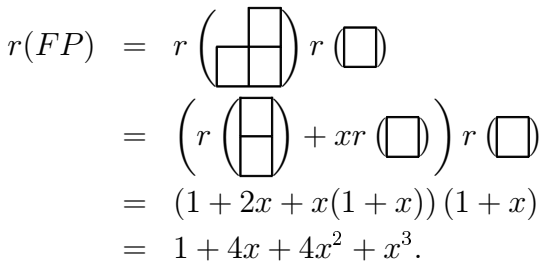
You are assigning 4 tasks T_1, T_2, T_3, T_4 to five colleagues C_1, C_2, C_3, C_4, C_5. Colleague C_1 has screwed up T_4 in the past and C_4 had the same experience with T_2. Colleague C_2 has threatened to quit if she has to do T_3 or T_4 again. You really, really need to keep C_2.

1. Fill in all the forbidden positions in the following assignment chart.

	C_1	C_2	C_3	C_4	C_5
T_1					
T_2				X	
T_3		X			
T_4	X	X			

2. Compute the rook polynomial for the chessboard consisting of the forbidden positions.

   $FP =$ forbidden positions.



$$\begin{aligned} r(FP) &= r\left(\quad\right) r\left(\quad\right) \\ &= \left(r\left(\quad\right) + xr\left(\quad\right)\right) r\left(\quad\right) \\ &= (1 + 2x + x(1+x))\,(1+x) \\ &= 1 + 4x + 4x^2 + x^3. \end{aligned}$$

   Therefore

$$r_1 = 4,\ r_2 = 4,\ r_3 = 1,\ r_4 = 0$$

3. How many ways can you assign the tasks to your colleagues, following the constraints?

c_i : Task T_i is assigned to a forbidden colleague:

$$\begin{aligned} S_0 &= 5 \cdot 4 \cdot 3 \cdot 2 = 120 \\ S_1 &= r_1 \cdot 4 \cdot 3 \cdot 2 = 96 \\ S_2 &= r_2 \cdot 3 \cdot 2 = 24 \\ S_3 &= r_3 \cdot 2 = 2 \\ S_4 &= r_4 = 0 \end{aligned}$$

$$E_0 = 120 - 96 + 24 - 2 + 0 = 46$$