

Rook Polynomials and Generating functions

- rook polynomials

```
> R[0,0] := 1;
```

$$R_{0,0} := 1$$

```
> R[1,1] := R[0,0] + x*R[0,0];
```

$$R_{1,1} := 1 + x$$

```
> R[1,2] := R[1,1] + x*R[0,0];
```

```
R[2,1] := R[1,2];
```

$$R_{1,2} := 1 + 2x$$

$$R_{2,1} := 1 + 2x$$

```
> r1 := R[2,1] + x*R[1,1];
```

```
R[2,2] := r1 + x*R[1,1];
```

```
R[2,2] := expand(R[2,2]);
```

$$r1 := 1 + 2x + x(1 + x)$$

$$R_{2,2} := 1 + 2x + 2x(1 + x)$$

$$R_{2,2} := 1 + 4x + 2x^2$$

```
> R[1,3] := R[1,2] + x*R[0,0];
```

```
R[3,1] := R[1,3];
```

$$R_{1,3} := 1 + 3x$$

$$R_{3,1} := 1 + 3x$$

```
> r1 := R[2,2] + x*R[1,2];
```

```
R[2,3] := r1 + x*R[1,2];
```

```
R[2,3] := expand(R[2,3]);
```

```
R[3,2] := R[2,3];
```

$$r1 := 1 + 4x + 2x^2 + x(1 + 2x)$$

$$R_{2,3} := 1 + 4x + 2x^2 + 2x(1 + 2x)$$

$$R_{2,3} := 1 + 6x + 6x^2$$

$$R_{3,2} := 1 + 6x + 6x^2$$

```
> r2 := R[3,2] + x*R[2,2];
```

```
r1 := r2 + x*R[1,2]*R[1,2];
```

```
R[3,3] := r1 + x*R[2,2];
```

```
R[3,3] := expand(R[3,3]);
```

$$r2 := 1 + 6x + 6x^2 + x(1 + 4x + 2x^2)$$

$$r1 := 1 + 6x + 6x^2 + x(1 + 4x + 2x^2) + x(1 + 2x)^2$$

$$R_{3,3} := 1 + 6x + 6x^2 + 2x(1 + 4x + 2x^2) + x(1 + 2x)^2$$

$$R_{3,3} := 1 + 9x + 18x^2 + 8x^3$$

- manipulating polynomials

```
> P := add(n*x^(n-1), n=0..10);
Q := expand(P^5);
coeff(Q, x, 14);
sort(Q);
```

$$P := 1 + 2x + 3x^2 + 4x^3 + 5x^4 + 6x^5 + 7x^6 + 8x^7 + 9x^8 + 10x^9$$

$$Q := 100000x^{45} + 450000x^{44} + 1210000x^{43} + 2519000x^{42} + 4472050x^{41} + 7104449x^{40}$$

$$+ 10382240x^{39} + 14198195x^{38} + 18372530x^{37} + 22657635x^{36} + 26746148x^{35}$$

$$+ 30331745x^{34} + 33152060x^{33} + 35016190x^{32} + 35819080x^{31} + 35544927x^{30}$$

$$+ 34261590x^{29} + 32107845x^{28} + 29275180x^{27} + 25985685x^{26} + 22467456x^{25}$$

$$+ 18928800x^{24} + 15542400x^{23} + 12437475x^{22} + 9698150x^{21} + 7366425x^{20} + 5448300x^{19}$$

$$+ 3921775x^{18} + 2745600x^{17} + 1867800x^{16} + 1233144x^{15} + 788865x^{14} + 488070x^{13}$$

$$+ 291405x^{12} + 167460x^{11} + 92323x^{10} + 48620x^9 + 24310x^8 + 11440x^7 + 5005x^6 + 2002x^5$$

$$+ 715x^4 + 220x^3 + 55x^2 + 10x + 1$$

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$$+ 715x^4 + 220x^3 + 55x^2 + 10x + 1$$

>