Disco I
Comments and Answers to Magma Session

1. Fixed Points

1. Remember that a …xed point is a card that is not moved during a shu - e. Make a table of expected number of …xed points and probability of getting a …xed point for several card shu - e sizes. Make a prediction for large deck size. Use the scripts `fixpnts.mgm` and `fixpntsR.mgm`.

group	total #…xed points	average #…xed points
S_3	6	$\frac{6}{6} = 1$
S_4	24	$\frac{24}{24} = 1$
S_5	120	$\frac{120}{120} = 1$
S_6	720	$\frac{720}{720} = 1$
S_7	5040	$\frac{5040}{5040} = 1$
large size	n!	1

group	total number of …xedpoints	average # of …xed point
S_3	6	$\frac{6}{6} = 1$
S_4	24	$\frac{24}{24} = 1$
S_5	120	$\frac{120}{120} = 1$
S_6	720	$\frac{720}{720} = 1$
S_7	5040	$\frac{5040}{5040} = 1$
large size	n!	1

1.

group	#decks with …xedpoints	prob of …xed point
S_3	4	$\frac{4}{6} = 0{:}66667$
S_4	15	$\frac{15}{24} = 0{:}625$
S_5	76	$\frac{76}{120} = :63333$
S_6	455	$\frac{455}{720} = :63194$
S_7	3186	$\frac{3186}{5040} = :63214$
large size		0:632 or so

2. Adjacencies

2. Do the same for adjacencies as in problem #1 using `adj.mgm`, `adjr.mgm`.

group	total #adjacencies	average #adjacencies
S_3	8	$\frac{8}{6}$ = 1:3333
S_4	36	$\frac{36}{24}$ = 1:5
S_5	192	$\frac{192}{120} = \frac{8}{5}$ = 1:6
S_6	1200	$\frac{1200}{720}$ = 1:6667
S_7	8640	$\frac{8640}{5040}$ = 1:7143
S_{52}	random sample	$\frac{9851}{5000}$ = 1:9702

group	# of decks with an adjacency	prob of adjacency
S_3	6	$\frac{6}{6}$ = 1
S_4	22	$\frac{22}{24} = \frac{11}{12}$ = :91667
S_5	106	$\frac{106}{120} = \frac{53}{60}$ = :88333
S_6	630	$\frac{630}{720} = \frac{7}{8}$ = :875
S_7	4394	$\frac{4394}{5040} = \frac{2197}{2520}$ = :8783

3. Runs of Three

3. Repeat #1 for runs of 3. You will have to modify the code of the other scripts.

group	total #runs of 3	average #runs of 3
S_4	8	$\frac{8}{24} = \frac{1}{3}$ = :33333
S_5	36	$\frac{36}{120} = \frac{3}{10}$ = :3
S_6	192	$\frac{192}{720} = \frac{4}{15}$ = :26667
S_7	1200	$\frac{1200}{5040} = \frac{5}{21}$ = :2381

group	# of decks with a run of 3	prob of a run of 3
S_4	6	$\frac{6}{24} = \frac{1}{4}$ = :25
S_5	28	$\frac{28}{120} = \frac{7}{30}$ = :23333
S_6	150	$\frac{150}{720} = \frac{5}{24}$ = .20833
S_7	958	$\frac{958}{5040} = \frac{479}{2520}$ = :19008

Somewhere around 0:2 so far.

Sample Code for runs of three

```
//// Counting runs of three in random shu - es of small decks
//Session …le
SetLogFile("run3.log": Overwrite := true);
//run of three counter using cycle notation
numrun := function(pi,n);
    s := 0;
    for i := 1 to n-2 do
        if (Abs((i+1)^pi-(i)^pi)+ Abs((i+2)^pi-(i+1)^pi)) eq 2
              then s := s+1; end if;
        end for;
    return s;
    end function;
//de…ne deck size, set up "shu - e group"
n := 5;
G := SymmetricGroup(n);
//average number of runs of three
nrun := 0;
for pi in G do
    nrun := nrun+numrun(pi,n);
    end for;
avgrun := nrun/#G;
print "total number of runs of three ", nrun;
print "avgrun = " , avgrun;
//probability of runs of three
r := 0;
for pi in G do
    if numrun(pi,n) gt 0 then r := r+1; end if;
    end for;
print "num of decks with runs of three ", r;
probrun := r/#G;
print "probrun = ",probrun;
 //End Session
UnsetLogFile();
```