Answers and Comments to Exercises

Exercise 1.5.2 and 1.5.3

Exercise 1.5.2

Exercise 0.0.1

1. Determine the FFPA form of ¼$^{i\,1}$ = (1; 2)(3; 4)(2; 3)(1; 2)(4; 5) and compare it with the FFPA form of ¼: ¼$^{i\,1}$ = (1; 3; 5; 4); they are the same

2. Determine the FFPA form of ¼$^{i\,1}$ = (3; 4)(4; 5)(2; 3)(1; 2)(2; 3) and compare it with the FFPA form of ¼: ¼$^{i\,1}$ = (1; 3; 5; 4); they are the same

3. Compute the inverse of

   (a) (1; 8; 9; 2; 7): (1; 7; 2; 9; 8)

   (b) (3; 6; 5; 4): (3; 4; 5; 6)

   (c) (2; 3; 9; 4; 5; 8): (2; 8; 5; 4; 9; 3)

   (d) (1; 8; 9; 2; 7)(3; 6; 5; 4): (1; 7; 2; 9; 8)(3; 4; 5; 6)

   (e) (1; 8; 9; 2; 7)(2; 3; 9; 4; 5; 8): (1; 7; 2)(3; 9)(4; 8; 5)

   (f) (3; 6; 5; 4)(2; 3; 9; 4; 5; 8): (2; 8; 6; 3)(4; 9)

4. T F (®¯)$^{i\,1}$ = ®$^{i\,1}$¯$^{i\,1}$: F

5. T F (®¯)$^{i\,1}$ = ¯$^{i\,1}$®$^{i\,1}$: T

6. Suppose ®¯ = ° in S_n.

   (a) T F ® = ¯$^{i\,1}$°: F

   (b) T F ® = °¯$^{i\,1}$: T

   (c) T F ¯ = °®$^{i\,1}$: F

   (d) T F ¯ = ®$^{i\,1}$°: T

Exercise 1.5.3

1. Determine $|AT_n|$: $n-1$

2. Write $(1;6;3;2)(4;7) \in S_8$ as a product of adjacent transpositions

$$(4;5)(5;6)(6;7)(1;2)(2;3)(3;4)(4;5)(5;6)(3;4)(4;5)$$

3. Write $(1;8)(2;7)(3;6)(4;5)$ as a product of adjacent transpositions.

$$\begin{aligned}
&(7;8)\\
&\cdot(6;7)(7;8)\\
&\cdot(5;6)(6;7)(7;8)\\
&\cdot(4;5)(5;6)(6;7)(7;8)\\
&\cdot(3;4)(4;5)(5;6)(6;7)(7;8)\\
&\cdot(2;3)(3;4)(4;5)(5;6)(6;7)(7;8)\\
&\cdot(1;2)(2;3)(3;4)(4;5)(5;6)(6;7)(7;8)
\end{aligned}$$

4. T F Each element of S_n can be written as a product of adjacent transpositions in a unique way. F

5. T F Two factorizations of an element of S_n into adjacent transpositions contain the same number of factors. T

6. Let $\#\text{atrans}(\pi)$ denote the number of adjacent transpositions appearing in an AT_n factorization of $\pi \in S_n$.

   (a) Determine the minimum value of $\#\text{atrans}(\pi)$. 0

   (b) Determine the maximum value of $\#\text{atrans}(\pi)$: $\sum_{k=1}^{n-1} k$ (which some students may recognize as $\frac{n(n-1)}{2}$ from a previous brush with induction).

   (c) Conjecture the average value of $\#\text{atrans}(\pi)$: Sampling or a heuristic argument should suggest $\frac{n(n-1)}{4}$ (which will be verified in Chapter 2 — see ??).