

Applied Mathematics I - Worksheet #10

Professor Broughton

Name: _____

Box #: _____

1. More on least squares

Recall the data from the last work sheet

i	x_i	y_i
1	3	3:20
2	4	4:60
3	5	7:3

and the vectors and matrix we introduced.

$$b = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}; e = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}; x = \begin{bmatrix} a \\ b \end{bmatrix}; \text{ and } A = \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ x_3 & 1 \end{bmatrix}$$

Also recall that we obtain x by solving the normal equations

$$A^t A x = A^t b:$$

1.a Using the data given above, write out the normal equations as a system of two equations in the two unknowns a and b :

2 Also recall that the x was chosen so that the sum of squares of the errors

$$e_1^2 + e_2^2 + e_3^2 = \|e\|^2 = \|Ax - b\|^2$$

was minimized.

2.a Using the data above. write out $kAx_j - bk^2$ as a function of a and b and sketch the function.

2.b Write out the system of equations you get by setting the partial derivatives $\frac{\partial}{\partial a} kAx_j - bk^2 = 0$ and $\frac{\partial}{\partial b} kAx_j - bk^2 = 0$

2.c Compare the two different systems.

2. Projections

From class we see that the matrix $P = A(A^tA)^{-1}A^t$ onto the column space of A so that we may solve:

$$Ax = Pb$$

exactly. Let's verify some properties of the projection operator P : Show your calculations.

$$2 \quad P^2 = P;$$

$$2 \quad P^t = P;$$

2 $b = Pb + (b - Pb)$ and b and $(b - Pb)$ are orthogonal. Illustrate this geometrically