Applied Mathematics I - Worksheet #9

Professor Broughton

Name:_____ Box #:_____

1. Least Squares

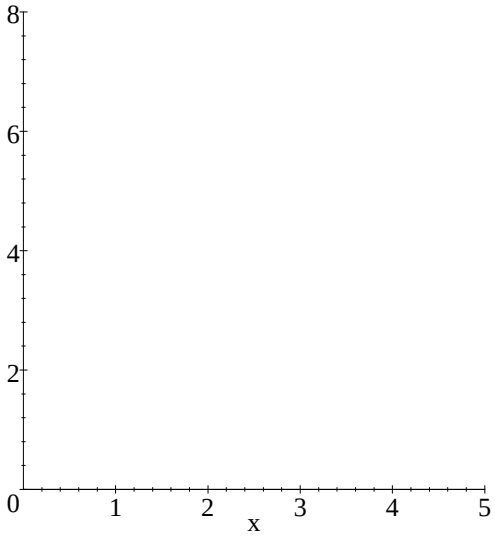
i	x_i	y_i
1	3	3:20
2	4	4:60
3	5	7:3

We would like to get a model that looks like

$$y_i = ax_i + b + e_i$$

Where a and b are the slope and intercept of the approximating line and e_i is the error. This can be extended to any number of data points.

1.a Plot the three points on the axes below and draw in your best approximating line. Estimate the slope and the intecept.



1.b Find the errors with your estimate and the sum of the squares of the individual errors

i	e_i
1	
2	
3	

$$e_1^2 + e_2^2 + e_3^2 =$$

2 Let $b = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$; $e = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$ and $x = \begin{bmatrix} a \\ b \end{bmatrix}$: Find the matrix A such that system of equations above is equivalent to:

$$b = Ax + e$$
$$Ax = b - e$$

3 Make a 3D sketch showing the range of A (a plane) and the three vectors Ax; b; and e

4 In class we will show that the error $\|e\|^2$ is minimized when e is perpendicular to the columns of A; i.e. $A^t e = 0$;

Show that this is the same as

$$A^t A x = A^t b.$$

5. Solve the equation for x both symbolically and numerically. What are the values of a and b? Compute the errors and the sum of errors again and compare the sum of squares to the earlier estimate.

i	e_i
1	
2	
3	

$$e_1^2 + e_2^2 + e_3^2 =$$

6 Look at the data in problem 55. Get the best linear and quadratic …ts you can. Plot the data itself, the linear …t and the quadratic …t simultaneously. Is the quadratic …t signi…cantly better? Write out the matrices A you used to get the linear and quadratic …ts.