Applied Math I - Test #2

Professor Broughton October 29, 1998

Name:_____ Box #_____

Instructions

- Answer all the questions directly on the test.
- Show all the necessary work and write your answers out neatly in English sentences. Use mathematical notation to express your answers, not Maple notation
- It is not necessary to use your computer to answer all of the questions but you can use it to obtain graphs, evaluate functions, solve equations, etc. If you use Maple be sure to say so by some sentence such as: Using Maple the above integral equals ....
- Recall that you may use notes that you can …t on one standard sheet of paper. On your computer you may start o¤ with one blank Maple worksheet only. Please hand in your sheet of notes with your test.

Question	Points
1	
2	
3	
4	
Total	

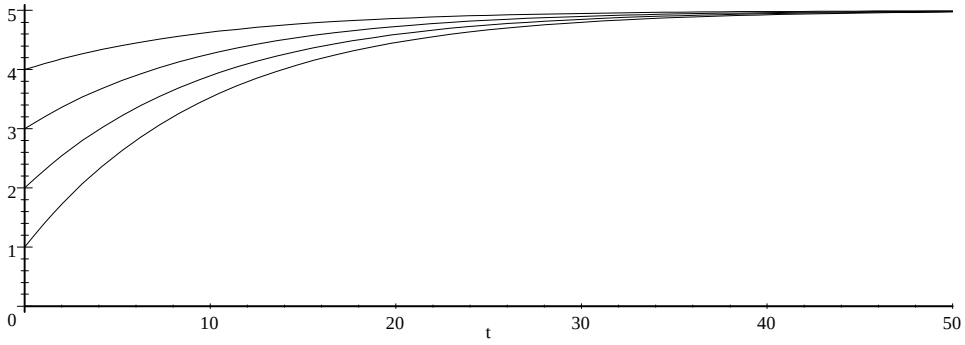
1.a The salinity of a 500 gal salt tank is being reduced by 50% by pumping in fresh water at 10 gal/min at the top and draining brine at an equal rate from the bottom. How long does it take to achieve the reduction. In your answer be sure to include a simple diagram summarizing the information, write out the DE to be solved,write out its solution, and a sketch a graph of the amount of salt over time.

1.b If the tank is only 250 gallons is the time to make a 50% reduction one half of what it was in the …rst part of the question?

2. A di¤erential equation has one of the following forms.

$$\begin{aligned} ay'(t) \text{ ¡ } y(t) &= K \\ y'(t) + by(t) &= K \\ y'(t) + by(t) &= \cos(2¼ ! t) \\ ay'(t) \text{ ¡ } y(t) &= \sin(2¼ ! t) \end{aligned}$$

The solution to the di¤erential equation for various initial conditions is given below. Find the correct equation and determine the parameters in the equation. $a; b; K$, and $!$ are all suppose to be positive numbers.



Suggestion: determine the stability and time constant information for the various equations and pick the best match.

3. Recall that our low pass filter D.E. has the form:

$$RCv_o'(t) + v_o(t) = v_i(t)$$

Suppose that $RC = 2$ and that the input signal is $3\cos(20t) + 2\sin(20t)$: Find a particular solution to the differential equation by the method of undetermined coefficients, showing your work. You may check your answer using dsolve.

4. The low pass …lter above is connected to a 9 volt battery with a switch. The battery is connected in such a way that the $v_i(t) = 9$when it is switched on. Suppose that $v_i(0) = 5$;and that the battery is switched on at $t = 5$.

4.a Write out a D.E. model for the circuit, using step functions or Heaviside functions as appropriate.

4.b Without solving, but using the convolution integral, write out the solution to the D.E. using the free response + zero initial condition forced response (homogeneous+particular) format.

4.c Solve the D.E. anyway you like and sketch the graph below for the10 seconds. In one or two sentences describe what happens to the voltage level during this time.