Applied Math I - Test #2

Professor Broughton October 29, 1998

Name:_____ Box #_____

Instructions

- Answer all the questions directly on the test.
- Show all the necessary work and write your answers out neatly in English sentences. Use mathematical notation to express your answers, not Maple notation
- It is not necessary to use your computer to answer all of the questions but you can use it to obtain graphs, evaluate functions, solve equations, etc. If you use Maple be sure to say so by some sentence such as: Using Maple the above integral equals ....
- Recall that you may use notes that you can …t on one standard sheet of paper. On your computer you may start o¤ with one blank Maple worksheet only. Please hand in your sheet of notes with your test.

Question	Points
1	
2	
3	
4	
Total	

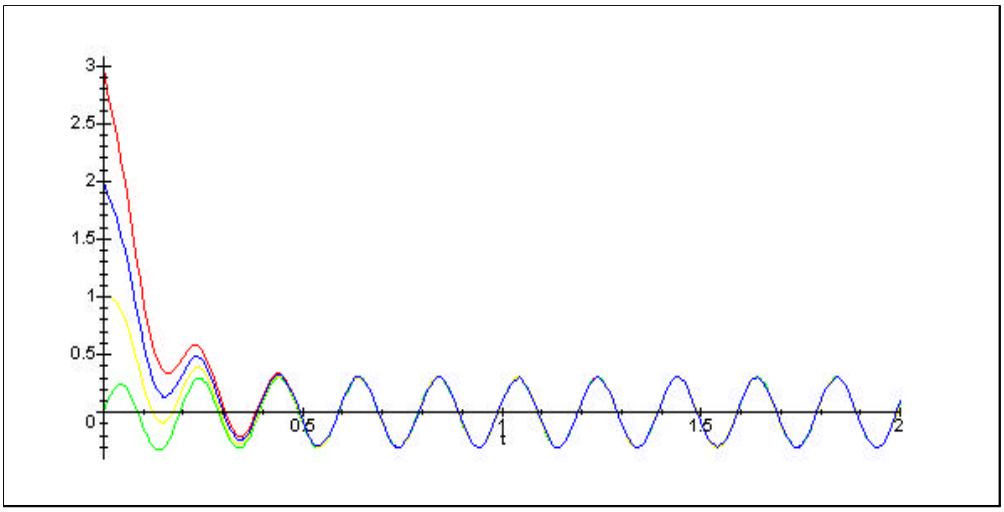
1.a A 500 gal salt tank is being ‡ushed out by pumping fresh water in at the top and pumping the brine out at the bottom, both at the same rate of 10 gals/min. Initially the concentration of the brine is 0.1 lbs per gallon. Determine the amount of fresh water required to ‡ush out the tank if we are to assume that we are only trying to reduce the salt level to 2% of the original amount. In your answer be sure to include a simple diagram summarizing the information, write out the DE to be solved,write out its solution, and a sketch a graph of the amount of salt over time.

1.b Compare your answer above to the amount required by draining and then re…lling the tank.

2. Four models are proposed for an unknown system

case	model	b(t)	Eigenvalue	Time Constant	Stable/Unstable
1	$0{:}1y'(t) - y(t) = b(t)$	$\sin(2\pi t)$			
2	$y'(t) - 0{:}1y(t) = b(t)$	5			
3	$0{:}1y'(t) + y(t) = b(t)$	$\cos(2\pi t)$			
4	$y'(t) + 0{:}1y(t) = b(t)$	9			

A signal b(t) in one of the forms above is fed into the system with varying initial conditions and the output y(t) measured. The results of 4 trial runs are graphed below.



Which model is the right one? Justify your answer. Suggestion: …ll in the stability and time constant information in the table and pick the best match.

3. Recall that our low pass filter D.E. has the form:

$$RCv_o'(t) + v_o(t) = v_i(t)$$

Suppose that $RC = .5$ and that the input signal is $\cos(10t) + 2\sin(10t)$. Find a particular solution to the differential equation by the method of undetermined coefficients, showing your work. You may check your answer using dsolve.

4. A 500 gallon salt tank contains 25 lbs of salt initially. Fresh water is pumped into the tank at 10 gal/min and drained o¤ the bottom of the tank at the same rate. After 1 hour the intake is changed to a brine solution with a concentration of .2 lbs/gal at the same rate for the next 4 hours. (Warning: In this problem convert every thing to minutes or hours!)

4.a Write out a D.E. model for the salt tank, using step functions or Heaviside functions as appropriate.

4.b Without solving, but using the convolution integral, write out the solution to the D.E. using the free response + zero initial condition forced response (homogeneous+particular) format.

4.c Solve the D.E. anyway you like and sketch the graph below for the …ve hours. In one or two sentences describe what happens to the salt level, including the …nal amount after 5 hours.