

## Calculus II - Test #3 Review

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### Instructions

- Answer all the questions directly on the test.
- For this test you may use Maple on your computer and/or your calculator. You may start with a blank Maple worksheet but no prepared worksheets.
- Show all the necessary work and write your answers out neatly in English sentences.
- Make reasonable simplifications.

Question	Possible Points	Points Obtained
1	40	
2	20	
3	20	
4	20	
Total	100	

## 1. Integrals

1.a Sketch the area contained between the two curves

$$\begin{aligned}x^2 + 9y^2 &= 9 \\x + y &= 1\end{aligned}$$

1.b Find all points of intersection between the two curves:

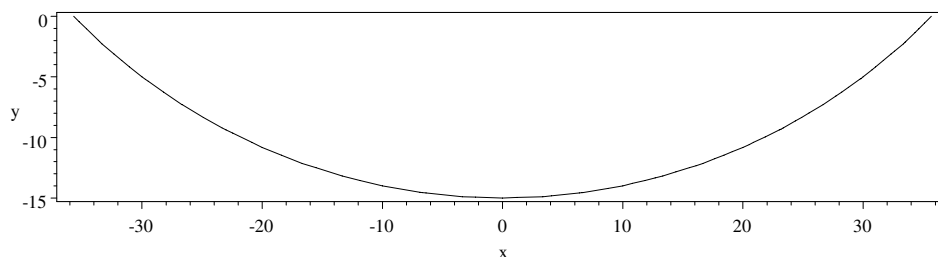
1.c Set up the integral(s) to compute the area between the two curves. You may integrate along either  $x$  or  $y$  axis:

1.d What is the area, to 4 decimal places. Say how you got the area.:

- 2 A waste lagoon is modeled as a lower portion of a sphere, and has a vertical cross-section as given in the figure below. The cross-section is given by the portion of the circle,

$$x^2 + (y - 35)^2 = 2500$$

lying below the  $x$ -axis. The dimensions are in feet.



- a What is the volume of water in the lagoon if height of the water is 5 ft from the top. Express the volume as an integral and then evaluate the integral. What a bout 10 ft from the top?

	Integral	Volume
5 ft		
10 ft		

1. b The lagoon is being pumped out at 25 cu ft per minute until the water level reduced to 10 feet below the top. How many hours does this take?

3 Compute these limits. Show all steps.

a

$$\lim_{x \rightarrow 5} \frac{x^2 - 25}{\cos(x - 5) - 1}$$

b

$$\lim_{x \rightarrow 1} \frac{\ln x}{x - 1}$$

c

$$\lim_{x \rightarrow \infty} \frac{x^3 + 3x}{e^{2x}}$$

d

$$\lim_{x \rightarrow \infty} \frac{\ln x}{x^2}$$

e

$$\lim_{x \rightarrow \infty} x^2 e^{-sx}, s > 0$$

4. Compute these integrals showing them as a limit first, then compute the limit. Make a sketch of the graph showing the infinite limit or the vertical asymptotes.

a

$$\int_0^{\infty} t^2 e^{-st} dt$$

b

$$\int_0^1 \ln x dx$$

c

$$\int_1^{\infty} x^p dx, p > 1$$

d

$$\int_{-\infty}^{\infty} \frac{dx}{1+x^2} = \int_{-\infty}^0 \frac{dx}{1+x^2} + \int_0^{\infty} \frac{dx}{1+x^2}$$

e

$$\int_0^2 \frac{dx}{(1-x)^2} = \int_0^1 \frac{dx}{(1-x)^2} + \int_1^2 \frac{dx}{(1-x)^2}$$