MA 112 - Calculus II
Worksheet # 8 - answers
Professor Broughton

Name:____________________ Box #:____________________

Due: Friday, Jan 31.
Read section 6.4 on arclength

Problem Statement: A metal yardstick, lying flat on the table, is pushed in at the ends so that it bends in the middle, as in the picture below. It is pushed in so that the ends are only 35.5, 35, 34.5 and 34 inches apart. We want to use a parabolic model to predict the deflection and then compare that to the measured deflection. This will help us test the reliability of our model.

1. When you receive the yardstick, two people of each team will place the metal yardstick on top of the wooden yardstick and push in the end the required amount. The others will measure the maximum deflection in the middle. Each team will report a deflection and the results recorded on the board. Record the results below,

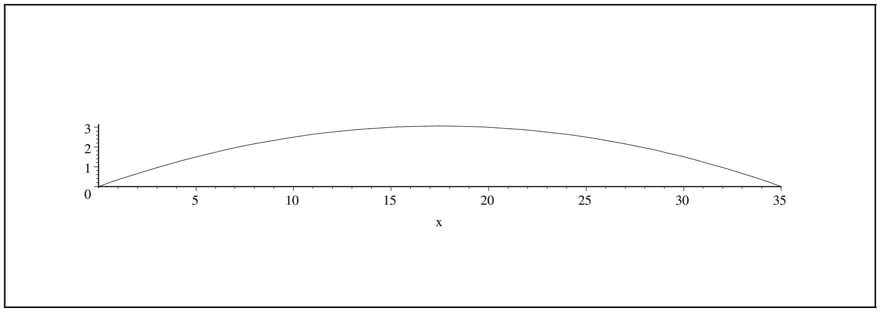
Team #	1	2	3	4	5	6
Amount pushed in e	0.5	1.0	1.5	2.0	0.5	1.0
Deflection h	2.5	4.0	4.75	5.25	2.74	3.89
Team #	7	8	9	10	11	12
Amount pushed in e	1.5	2.0	0.5	1.0	1.5	2.0
Deflection h	4.75	5.46	2.6	3.75	4.5	5.25

2. For each amount pushed in find the average deflection. Thus the results for Teams 1,5, and 9 could be averaged.

3. While waiting for the ruler Consider the following modeling problem. Assume that the ruler follows the following curve:

$$r(x) = Ax(L_e - x) \tag{1}$$

where $L_e = 36 - e$. and $r(L_e/2) = h$. Find A in terms of h and e.



$$h = A(18 - e/2)(18 - e/2)$$
$$A = \frac{4h}{(36 - e)^2}$$

4. For each of the amounts pushed in find the equation of the ruler and the computed arclength of the ruler model

Amount pushed in e	0.5	1.0	1.5	2.0
Average Deflection h	2.613	3.880	4.667	5.353
coefficient A	8.294×10^{-3}	1.267×10^{-2}	1.568×10^{-2}	1.858×10^{-2}
Arclength	36.007	36.115	36.116	36.130

Sample calculation:

$$A = \frac{4 \times 2.613}{(36 - 0.5)^2} = 8.2936 \times 10^{-3}$$

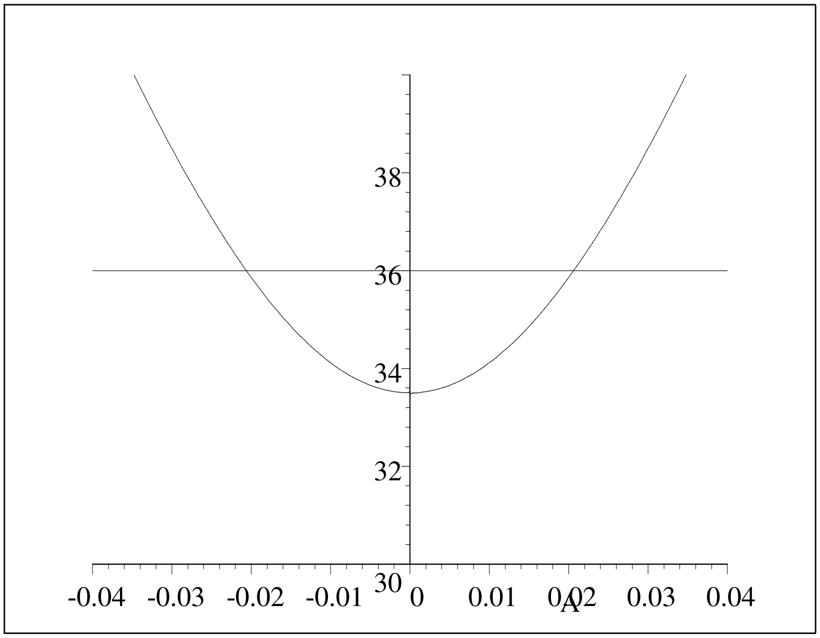
$$r(x) = \frac{8.2936}{1000} x(35.5 - x)$$
$$r'(x) = \frac{8.2936}{1000}(35.5 - 2x)$$

$$
\begin{aligned}
AL &= \int_0^{35.5} \sqrt{1 + \left(\frac{8.2936}{1000}(35.5 - 2x)\right)^2} dx \\
&= 36.006
\end{aligned}
$$

5. Suppose that the ruler is pushed in 2.5 inches. Predict the deflection Suggestion leave A as a variable in equation 1 and the compute the arclength as a formula in A. Set the quantity equal to 36 and solve for A and then h.

$$
\begin{aligned}
r(x) &= Ax(33.5 - x) \\
r'(x) &= A(33.5 - 2x)
\end{aligned}
$$

$$
\begin{aligned}
AL &= \int_0^{33.5} \sqrt{1 + (A(33.5 - 2x))^2} dx, \text{ (set } u = 33.5 - 2x) \\
&= \frac{1}{2}\int_{-33.5}^{33.5} \sqrt{1 + A^2u^2} du, \text{ (set } v = Au) \\
&= \frac{1}{2A}\int_{-33.5A}^{33.5A} \sqrt{1 + v^2} dv \\
&= \frac{1}{2A}\left(16.75A\sqrt{4.0 + 4489.0A^2} - 0.693\,15 + \ln\left(67.0A + \sqrt{4.0 + 4489.0A^2}\right)\right)
\end{aligned}
$$



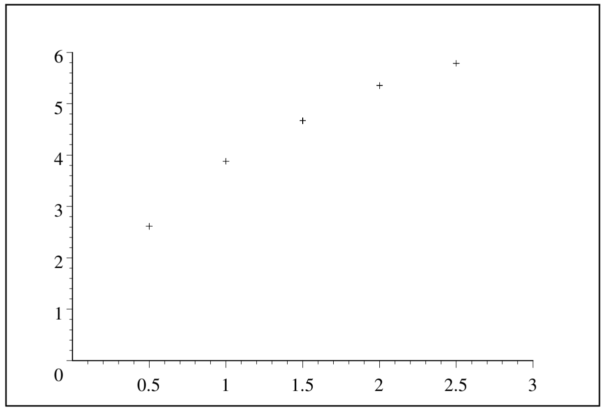
Solve

$$\frac{1}{2A}\left(16.75A\sqrt{4.0+4489.0A^2}-0.693\,15+\ln\left(67.0A+\sqrt{4.0+4489.0A^2}\right)\right)=36$$

numerically to get

$$\begin{aligned} A &= 2.062\,0\times 10^{-2} \\ h &= \frac{A}{4}(L-e)^2 \\ &= \frac{2.062\,0\times 10^{-2}}{4}\left(33.5\right)^2 \\ &5.785 \end{aligned}$$

Fianlly a plot of e vs h



Plot of e vs h